A Synthetic Eddy Method for generating inflow conditions for LES

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Introduction to Synthetic Turbulence

• A need specific to LES and DNS
• A wide range of applications
  – Initialisation of flow fields
  – inlet boundary conditions, embedded LES
  – Forcing at the interface of hybrid RANS/LES
• How to generate realistic turbulence?
  – Most efficient methods are not industrial (database rescaling, POD)
  – Synthetic generation of turbulence from RANS statistics
    • random method
    • spectral method (Lee et al. Physics of Fluids 1992)
    • digital filtering (Klein et al. Journal of Computational Physics, 2003)
Synthetic Eddy Method (SEM) Principle

- A turbulent flow is a superposition of coherent eddies

- Each eddy is described by a shape function $f(x)$ with compact support

  $[-r_x; r_x] \times [-r_y; r_y] \times [-r_z; r_z]$ 

- Each eddy has a random position $(y_i, z_i)$ on the inlet plane

- Each eddy is convected through the inlet plane until it is not active anymore ($x_i > r_x$), then a new one is regenerated at $x_i = -r_x$
Synthetic Eddy Method (SEM)

- **Formulation**

  - The eddy signal
    
    \[ v_j(x,t) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \varepsilon_i f_j(x - x_i(t)) \]

    - \( N = \text{number of vortices} \)
    - \( \varepsilon_i = \pm 1 \)

  - Rescaling
    
    \[ u_i(x,t) = U_i(x) + a_{ij}(x) \ v_j(x,t) \]

    - \( (a_{ij}(x)) \) Choleski decomposition of \( (R_{ij}(x)) \)
    - (by construction, not computed)
Synthetic Eddy Method (SEM)

- **Signal characteristics**
  - Mean velocity and Reynolds stresses profiles
  - Autocorrelation function and energy spectrum imposed by shape function
    \[ R_{11}(r) = \langle f_1(x) f_1(x + r) \rangle \]
  - Low computational cost due to limited number of eddies
  - Divergence free inlet condition with divergence free shape functions

- **Free parameters**
  - Length and time scales (*tuning is objective of current work*)
  - Shape function of eddies (*tuning is objective of current work*)
Spatially Decaying Isotropic Turbulence

• Parameters
  – $U_c = 20$, $k = 3/2$, $\nu = 10^{-4}$

• Mesh
  – $L_x \times L_y \times L_z = 6\pi \times 2\pi \times 2\pi$
  – $N_x \times N_y \times N_z = 192 \times 64 \times 64$ cells

• Numerical Procedure
  – Finite volume unstructured incompressible code: Code_Saturne
  – Smagorinsky model with Cs = 0.18

• Tests
  – 1) Compare methods, 2) Size of eddies, 3) Shape of eddies
HIT: Methods inlet

1) ... Random method
2) ___ SEM with 3D isotropic spots \( f(r) \) gaussian
3) - - - DIG (gaussian filter)
4) -.-.- SPE (\( k^2 \exp \left(-k^2/k_0^2\right) \) spectrum)

\[
\begin{align*}
\begin{bmatrix}
-k^2 & 0 \\
0 & -k^2
\end{bmatrix}
\end{align*}
\]

3D spots approach

\( f(r) \)

\( E(k) \)

\( k \)

\( k_0 \)
HIT: Methods results

\[ S_k \]

\[ n_x \]

\[ k \]

\[ E_{11}(k_1) \]

\[ k_1^{-5/3} \]
HIT: Eddies Size

1) … Random method
2) - - - SEM with 3D isotropic spots (small)
3) ___ SEM with 3D isotropic spots (medium)
4) - .- . - SEM with 3D isotropic spots (big)
HIT: Eddies Shape

1) ... SEM with 3D isotropic spots (gaussian)
2) - - - SEM with 3D vortices (gaussian)
3) ___ SEM with 3D isotropic spots (model function)
HIT: Shape results

$E_{11}(k_1)$

$\kappa_1^{-5/3}$

SEM model shape function

SEM Spot Gauss.
SEM Vortices
SEM Spot Model Func.
Channel Flow Re*=395

- **Mesh**
  - $Nx \times Ny \times Nz = 160 \times 30 \times 30$ cells
  - $\Delta x^+ \approx 60$, $\Delta z^+ \approx 40$, $\Delta y^+_{\text{min}} \approx 1$
  - inlet/outlet b.c.

- **Numerical Procedure**
  - *Code_Saturne*
  - Smagorinsky model $C_S = 0.065$
  - Precursor periodic channel flow, statistics are fed into synthetic methods
  - All inflow data have same mean and Reynolds stresses profiles
Channel: Methods results

Q Contours

RAND

PREC

SPECL02

SEML02
Channel: Methods results

\[
\frac{c_f}{c_{f_0}}
\]

\[
\bar{u}^2
\]

\[
\bar{uv}
\]
Channel: Length-scale results

SEML01 Small spots

SEML02 Medium spots

SEML04 Large spots
Channel: Shape results

- SEM Medium Spots (x x x)
- SEM with 3D Vortices (___)
- SEM with Wall Model (---)

In the centre

\[ l = \max(\Delta z, k^{3/2} / \varepsilon) \]

At the wall

Wall model approach
Conclusions & Future Works

• SEM gives good results compared to other synthetic methods

• Influence of the length-scale of the inflow

• How to improve the SEM
  – Better modelisation of the large scales?
  – Better modelisation of the small scales, wavelet analysis