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Flow over a 2D hill: reference solutions for $k - \varepsilon$ and Second Moment closure Turbulence models

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Abstract:

1 Introduction

ERCOFTAC SIG 15 organises yearly workshops for the evaluation of numerical simulation procedures for refined modelling of turbulence flows. This activity, initiated some 15 years back through IAHR and later organised jointly with ERCOFTAC, was boosted by the 1995 Workshop in Karlsruhe which concluded the EC funded SCIENCE programme on « Databases and Testing of Calculation Methods for Turbulent Flows » resulting in a database of 70 test cases (see ERCOFTAC Web page).

Theses 2 day meetings allow to discuss numerical methods and turbulence models, but on a number of occasions definite conclusions could not be reached, as different groups using the same turbulence models seldom arrived at identical results. Inlet conditions, convection schemes, mesh refinement were questionned, and promises to rerun the calculations were made, but unkept, understandably as all this activity is not funded. For high Re flows, some even whether it is possible at all to reach indentical solutions with different codes, due to the different grid arrangements and most of all different, code specific, implementations of wall functions.

To this purpose, 2 undergraduate students were given the task of computing the same test case with different codes at their disposal; one in at EDF R&D Paris, and one at UMIST Manchester. Until the final stage, each student was asked to establish his own "best converged" solution, using for this assessment text-book rules such as checking grid independance, convection schemes, inlet conditions ... Only in the end did they actually compare their solutions which were found to coincide, to our relief but also surprise given the previous difficulties exhibited by the workshops.

Of the 4 codes used in this exercises 2 are major commercial codes and 2 "in house " but well validated codes. All use finite volumes, and in this case only structured grids (one of them staggered, 3 others collocated). The names of the codes and their specific features are irrelevant since the purpose here is to establish "a reference solution" for each turbulence model, not to compare advantages and weaknessess of these codes.

This reference solution represents the true solution to e.g. the k-ε model, independantly from the numerical methods or even the specific grids (as long as they are fine enough). This reference solution is intended to be included in the database alongside the experimental data from which it inevitably differs. It can then be used in validation procedures of new codes. The exercise then consists in recovering this reference solution for a given model, and not to match the experimental data as close as possible. Only when this is achieved can discussions on "improvements" of the turbulence models be considered.

The chosen test case is the flow over a single hill. It is a fairly simple geometry, yet the predictions size of the recirculation originating from the top of the hill, was found to be very dispersed even for users of identical

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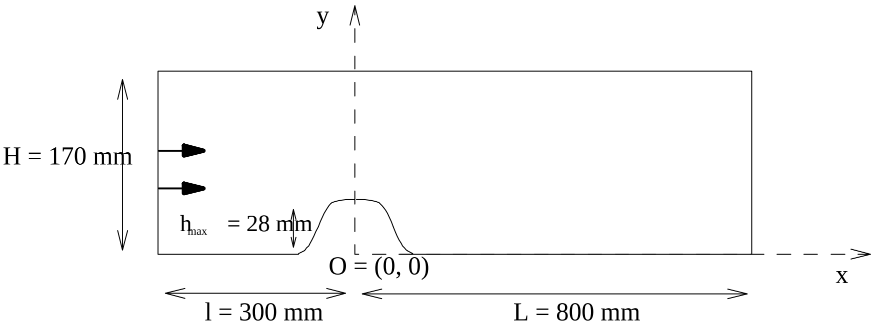


Figure 1: 2D single hill flow : Dimensions of the domain

models ([?]). It was considered at the 4th Workshop, organised by University of Karlsruhe, IFH, April 1995, (proceedings: ftp://ftp-ifh.bau-verm.uni-karlsruhe.de/workshop_sci_95/Proceedings). Streamlines obtained at the time of the workshop are shown on Fig. 1. General conclusions on the performance of the various models were difficult to find, partly because there was a large amount of data to be compared and because quite different results were obtained with nominally the same turbulence models by different contributors. However some general trends could be drawn from the workshop for the k-ε model and the Reynolds-stress model (RSM).

For this case, the more complex Reynolds-stress model is not consistenly better than the k-ε model concerning the practical relevant mean quantities. It performs a bit better for the predictions of the turbulence quantities.

2 Presentation of the case

The experiments are due to Almeida et al. (1993). The channel height is 170 mm and the maximum height of the hill hmax = 28 mm and the length 108 mm. The Reynolds number based on hmax and the channel centre-line velocity (upstream of the hill) is 60,000. The measurements were taken in the vertical centre-plane of the tunnel, but since this was only 200 mm wide (compared to the height of 170 mm), some deviations from two-dimensionality of the flow in this plane cannot be ruled out even though this was assumed in the calculations.

As the opposite channel wall is 6 hill heights from the bottom wall, it has relatively little influence on the flow over the hill. A fairly large recirculation zone having a length of 4.4 hill heights develops.

The fluid is water and the properties of the flow for the validation are :

Kinematic viscosity:	$\upsilon = 1 \times 10^{-6} m^2/s.$
Density:	$\rho = 1\ kg/m^3$
Mean centreline velocity at inlet:	$U = 2.147 m/s.$
Reynolds number:	$\mathrm{Re}_{h\,\max} = 60000$

Table 1: 2D single hill: characteristics of the flow

The available experimental profiles are referenced by their number, as described in table ?.

Number of the profile	-3	-2	-1	00	01	02	03	04	05	06	07	08	09	10	11
x (in mm)	-300	-50	-20	0	30	50	70	90	120	134	150	185	225	300	500

Table 2: Position of the profileskey

To illustrate the position of these profiles with respect to the flow caracterisitics, the streamlines corresponding to what we believe to be "reference solutions" for the k-ε and Second Moment Closure are shown in figure 2??. These results will be commented further.

3 Assessments of the reference calculation

The 4 codes will be designed by the letter C, E, F and S. Grid refinement effects were tested independendly with E and F.

Grids used for F is described in 3

code F	medium	fine
Number of cells	16000	64000
Dimensions	200*80	400*160
Mesh size in the x direction	1.6 mm $\leq \Delta x \leq$ 30 mm	0.8 mm $\leq \Delta x \leq$ 15 mm
in the y direction	0.1mm $\leq \Delta y \leq$ 1mm	0.1mm $\leq \Delta y \leq$ 0.3mm
y^+ (at inlet)	58	55

Table 3: 2D single hill: GAMBIT grids

Notice that when doubling the mesh, the first near wall cell was kept sufficiently large to remain well in the Log. layer.

The mesh generated for code F is fairly clustered near the lower wall (figure) and hardly any differences could be noticed in the results between Mesh 1 and Mesh 2.

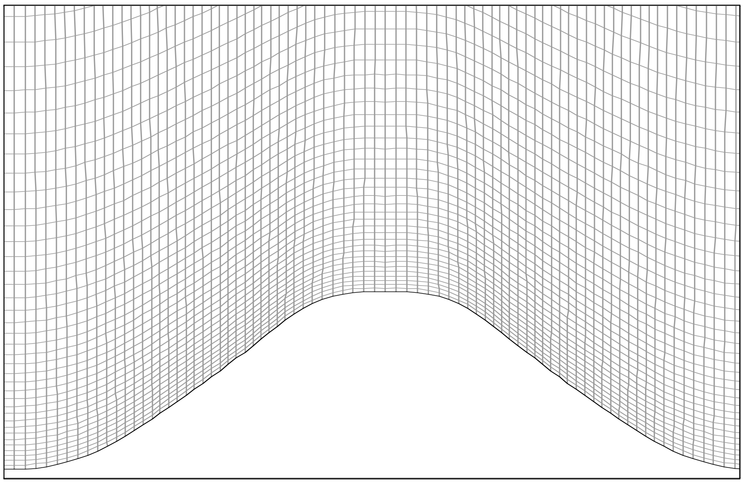


Figure 2: Mesh Refinement near the walls for F code

On the other hand, the user with code E generated a nearly constant grid, with 3 meshes of 4,000 ; 16,000 and 64,000 nodes, so that the effect of mesh refinement is much more visible (the near wall cells with code E & 64,000 nodes are nearly identical to code F with the 16,000 nodes grid).

code E	coarse	middle	fine
Number of cells	4000	16000	64000
Dimensions	100*40	200*80	400*160
y^+ (at inlet)	104.	52	26

With code F, no difference between the 3 meshes is found until the top of the hill where it is seen that the vertical velocity component increases while the axial component maximum is decreased, both coming very close to the experiment.

The effect of mesh refinement is better illustrated with code F, on figure 3 ?, at the top of the hill, x=00, and just beyond reattachement, x=05 (before the summit is no effect). From the U and k profiles it is seen that most of the variation originates from the variation ot the height of the first near wall cell. This explains why with code F and keeping this size constant hardly any change was observed. The variation of the V profiles are more important, and spread further away from the wall. This can be explained by the fact that as the near U velocity decreases with mesh refinement, the vertical deflection of streamlines must increase in order to conserve mass. Due to the elliptic nature of pressure, this effect speads much further in the vertical direction.

The maximum of k at x=05 is seen to be sensitive to mesh refinement. This was also where some differences were noted between the medium and fine mesh with code F. It is not surprising since the production of turbulence is very sensitive to the mean velocity gradients, and moreover the profile of k itself is clearly more sensitive to discretisation than the quasi linear mean velocit profile.

3.1 Inlet conditions

Effect of inlet conditions, imposed at 3 hill heights upstream of the hill summit, were tested with code F (at Manchester). Using either a constant profile for k based on turbulence intensity, $k = \frac{3}{2}(UI)^2$, or one fitted from the experimental data made no difference. Imposing a constant dissipation profile, corresponding to the average of ε across a fully developped channel flow, led to a dramatic overestimation of k on the hill summit. Using the lenghtscale relation $L = C_\mu^{3/4} k^{3/2}/\varepsilon = \kappa y$ led to a good agreement of velocity and k profiles with experiments at the hill summit.

With code E (Paris), analytical profiles based on a compilation of DNS and LES results of channel flows (AGARD) were defined as follow:

$$
\begin{aligned}
U^+ &= \left(\frac{1}{\kappa} * \ln(1 + 0.4 * y^+) + 7.8 * (1 - \exp(-\frac{y^+}{11}) - \frac{y^+}{11} * \exp(-\frac{y^+}{3}))\right) && (1) \\
k^+ &= 0.07 * {y^+}^2 * \exp\left(-\frac{y^+}{8}\right) + 4.5 * \left(1 - \exp\left(-\frac{y^+}{20}\right)\right) * \frac{1}{\frac{4*y^+}{\mathrm{Re}_*} + 1} && (2) \\
\varepsilon^+ &= \frac{1}{0.41((y^{+^4} + 15^4)^{0.25}} && (3)
\end{aligned}
$$

A reference u_* is provided from the experimental data, which value is : 0.079 m/s, in accordance with usual correlations for friction coefficients or head losses.

These profiles matched the experimental data at inlet and hill summit quite well. In a later stage these same profiles were also adopted for code F, and yieled a small but visible improvement at the hill summit. These inlet conditions were subsequenly used for all results and all codes shown herein. Such refined profiles would actually only be usefull for testing low Re models, but were convenient here to ensure that the same inlet profiles were given for the 4 codes whatever the grids.

As concerns the SMC simulations, the same analitycal k and ε profiles were used. Surprisingly no differences were found at the hill summit when using either isotropic or the algebraic stress model assumptions for anisotropy. It seems that the 3 hill height after the inlet is sufficient to recover developped stress profiles while starting from the crude isotropic turbulence assumption.

3.2 Reference solution for the $k - \varepsilon$ and SMC models

The constants of the $k - \varepsilon$ model are given the usual values (Launder et Spalding (1974)):

C_μ	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$
0.09	1.0	1.3	1.44	1.92

Table 4: k-e model constants

The SMC uses the standard Rotta return to isotropy pressure strain term, and a linear izotropisation of production, with also standard values C_1=1.8. and C_2=0.6. For the turbulent transport terms, it should be noted that instead of the Daly & Harlow model used in codes E and G, code F uses an simple eddy viscosity with a Schmidt number of $\sigma_k = 0.82$ (Lien and Leschziner [98])

The $k-\varepsilon$ model results are shown on figure ?? The agreement of the simulations with each other is astonishingly good, and we recall that code F was run in Manchester, while E and G were used in Paris with a different grid and different user.

The agreement with the experimental data is also excellent, including for the V component which is very small. It is a common "pun" to say that an experiment is validated by numerical similations, but we believe that the data of Almeida et al is extremely reliable and highly recomendable for code validation. The fact that both U and V profiles are in good agreement vith the 2D simulations is a proof that the experiment is free from 3D effects, in opposition to the statement (at the time of the Workshop) that the spanwise dimension of the rig may have been too small.

The slight underestimation of the reverse flow may be attributed to use of wall functions in the recirculation bubble. Low Re models applied to a backward facing step are known to correct this feature.

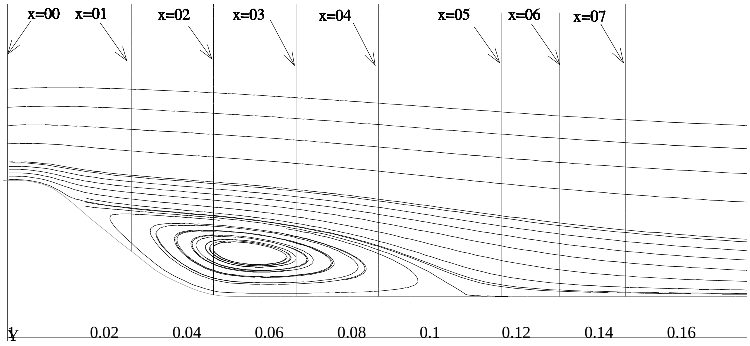
The overestimation of k at the hill summit is a classical defect of the $k-\varepsilon$ model which overestimates the production in plan strain as occurs along the uphill part ot the flow.

Figure now shows results for the SMC. As might have been expected, this model cures the overestimation of production during the "compression" uphill. On the summit the k profiles are in much better agreement with the experiment. Yet the maximum of velocity is visibly overestimated, resulting in an underestimation of the vertical deflection of the flow. It is a if friction on the uphill part had been under estimated. Note that this friction-deflection link is consitent with the previous dicussion on mesh refinement.

However, by superimposing the streamlines for both models shown on figure ??, we observed that seperation occurs exactly at the same point, about 0.015 m downstream of the summit. Excluding the recirculation bubble, the streamlines of both models are identical down to x=03. This means that the slight error in the V profile at the summit for the SMC, does not influence the downstream developpment. The main difference between models is in the recircultation bubble, which is much more elongated for the SMC, with a centre shifted from 0.05 to 0.08m The kink in the streamlines at reattachement point can be cured by a correction to the disspation (Yap correction) as discussed in detail by Hanjalic, but since this option was not available in all codes, it has not ben used to define the reference solution.

The V profiles at x04 clearly show that the structure of the bubble is better represented by the $k-\varepsilon$ model, this is also confirmed on other profiles wich will be availble on the Website. From these it can also be observed that the SMC dramatically underestimates the growth of the kinetic energy along the separated shear layer, even more than can be seen from the X04 profile. The $k-\varepsilon$ model is doing only slightly better, but it wold be interesting to test if this is not just "by chance", since this model starts at the summit with a severe overestimation of k. For a fair comparison it would be interesting to reset k and ε at the summit at the values obtined with the SMC, and to see to what extent this affects the recirculation bubble.

4 Conclusion



Streamlines behind the hill with k-ε model

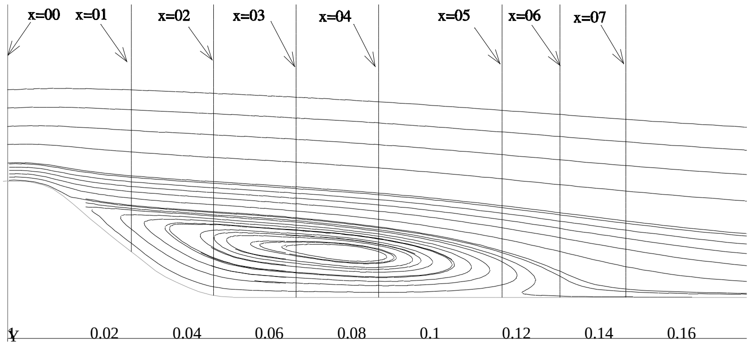


Figure 3: Streamlines behind the hill with RSM model

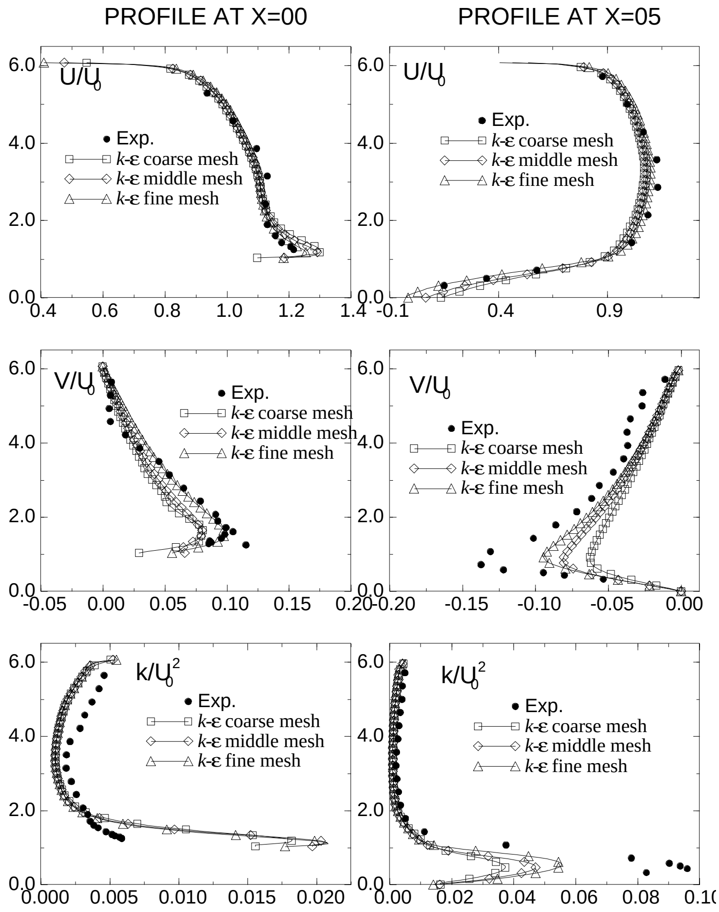


Figure 4: Profiles at x=00 and x=05 for code E with k-ε model on three different meshes

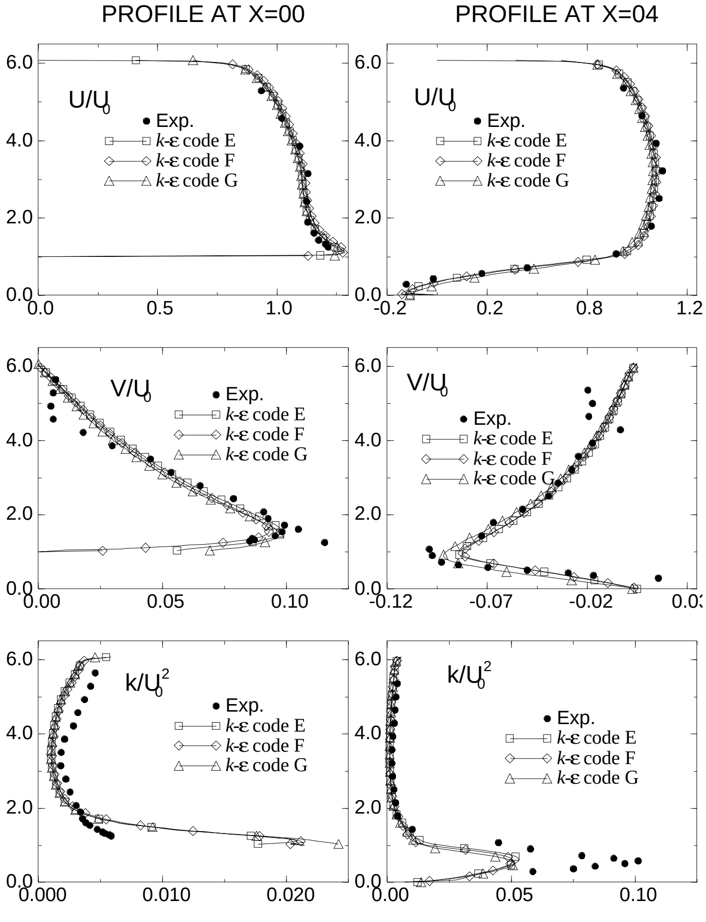


Figure 5: Profiles at x=00 and x=04 for codes E, F and G with k-ε model

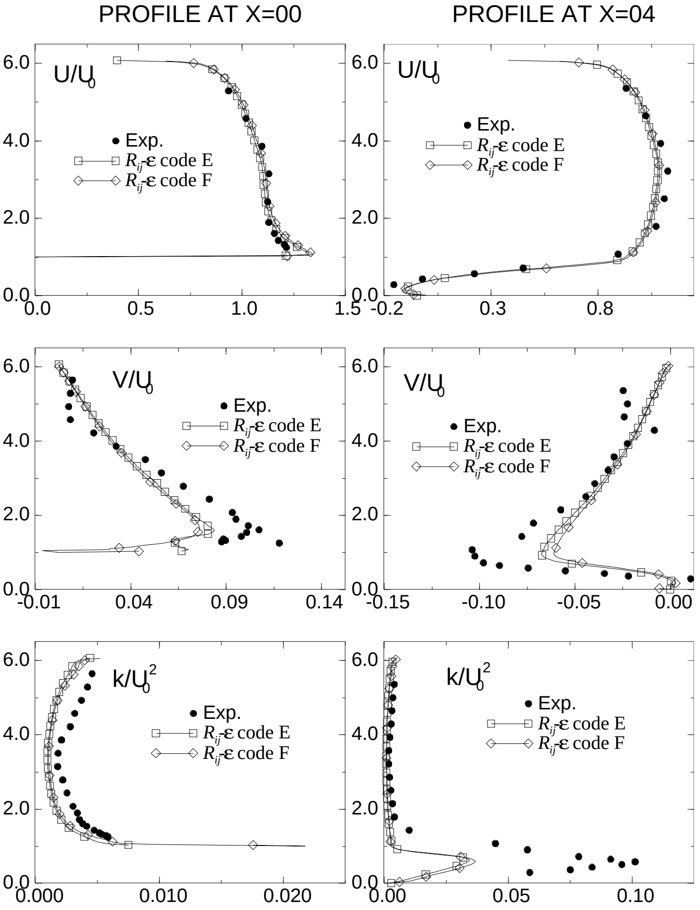


Figure 6: Profiles at x=00 and x=04 with RSM model