# Low-Reynolds-Number Subgrid-Scale Models

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**Abstract**

Subgrid-scale models are derived for large-eddy simulations in the limit of low Reynolds number, or, equivalently, resolution approaching that required for full resolution of the simulated turbulent flow. The models are constructed from standard forms of the dissipation spectrum in a manner analogous to that used to derive the classical Smagorinsky-Lilly model from the inertial range spectrum. Practical methods for computing the subgrid-scale eddy viscosity are described, together with examples of the effects of using such models in a real simulation.

## 1 Introduction

Computational fluid mechanics, like most science-based disciplines, proceeds only occasionally through revolution, and the rest of the time through a slow evolution that builds on the soundest elements of what is known. The revolution in our understanding of turbulence that occurred in 1941 was of this character, with the inertial range spectrum of Kolmogorov (1941 a, b) and others being found in turbulent flows that cannot strictly be described as homogeneous, isotropic or of sufficiently high Reynolds number to possess a long inertial range.

Large-eddy simulation of turbulence has also benefited and suffered from the well-known 'embarrassment of success' of the $k^{-5/3}$ spectrum. The original subgrid scale model of Smagorinsky (1963) and Lilly (1966, 1967) was firmly based on dimensional arguments and integral relations coming directly from Kolmogorov's ideas. The fact that this type of model has found such wide application in LES is associated with this strong foundation.

Real improvements in subgrid-scale modelling have been difficult to uncover. Of a number of innovative ideas introduced during the 1970s and '80s, the dynamic subgrid-scale model of Germano *et al.* (1991) is considered the most promising currently. It formalises the recognised need for the strength of the subgrid coefficient to vary not only from flow to flow but within a single flow (and also possibly with time), extracting the variable coefficient from the simulated flow in a rigorous manner. The dynamic subgrid concept can be applied to a number of base subgrid models, the most common being of the Smagorinsky-Lilly type. The concept does have clear weaknesses, notably the identification in its simplest forms of the time-scale of the model term with a time-scale extracted from the resolved motions, and its assumption that the underlying form of the model, including the value of the coefficient, cannot change with scale. This is not credible when the simulation cut-off (the smallest resolved scale) lies in a range where the subgrid drain is no longer independent of scale — for instance when the cutoff lies in the dissipation range.

Dynamic models have been applied with some success to LES of transitional flows, since the necessary decrease in the subgrid coefficient as the cutoff moves into the dissipation range is partially quantified by measuring the level of energy transfer in the smallest resolved scales. However, the commonest dynamic models fix the coefficient locally according to the energy drain from a test scale that is twice the grid scale, and in the dissipation range a change of scale by a factor of two can make a major difference to the turbulence dynamics. So although a dynamic model will (correctly) predict lower levels of the subgrid coefficient than a classical Smagorinsky-Lilly model when the cutoff is in the dissipation range, it may be expected still to overestimate the value. Even those who use dynamic models in this regime with some success (Fatica *et al.* 1994) recognise that there is an inconsistency in the underlying philosophy, since the coefficient must change systematically with scale in the dissipation range.

The principal purpose of this paper is to formulate rigorously, on the basis of hypothesised dissipation spectra, the manner in which the subgrid coefficient changes when the cutoff occurs at dissipation scales. While this may prove useful for dynamic modelling in simulations of transitional flow, it has a broader application to low-Reynolds-number subgrid models in general and will be presented in this context.

Any attempt to perform and critically assess large-eddy simulations of low-Reynolds-number or low-Peclet-number flows rapidly leads to the conclusion that the classical models are far too strong in this regime. The Smagorinsky-Lilly type of model, whose coefficient is proportional to the square of the grid scale, vanishes too slowly, and predicts non-zero values of the subgrid-scale eddy viscosity even in completely resolved simulations. Large-eddy simulations of transitional flows using unmodified models normally founder, failing to find transition in the correct place or at all, because of the greatly overestimated eddy viscosity. A number of workers have recognised these deficiencies. Horiuti (1986) computed the late stages of transition, and Deschamps (1987) a transitional channel flow, both finding that traditional modelling was too dissipative. Piomelli *et al.* (1990) reviewed the use of LES for transitional wall-bounded flows, and showed that such simulations are viable with suitably scaled or damped forms of the models. Ducros and Comte (1994) have recently reported an algebraic filter method for reducing the strength of the velocity-structure-function subgrid-scale eddy visosity for the prediction of boundary-layer transition. Grötzbach (1986) has pursued the study of low-Peclet-number LES in depth, formulating well-founded methods for subgrid-scale modelling in such flows.

The methods described in this paper are distinct from these workers' approaches. They have been applied successfully to LES of bypass transition by Yang and Voke (1993). Comparitive results given later are from the thesis of Zhao (1994), who has tested a number of subgrid modelling innovations in the context of low-Reynolds-number channel flow.

## 2 The Smagorinsky-Lilly Model

The model is deduced through a dimensional argument using the mesh scale $h$ and a velocity scale related to the resolved strain rate. Using common notations, the strain rate tensor $\bar{s}$ is the symmetric part of the velocity deformation,

$$\bar{s}_{ij} = (u_{i,j} + u_{j,i})/2 \ , \tag{1}$$

and the strain scalar is then defined as

$$\bar{s}^2 = \bar{s}_{ij}\bar{s}_{ij} \ . \tag{2}$$

The subgrid-scale eddy viscosity is

$$\nu_s = C_s h^2 \sqrt{2\bar{s}^2} \tag{3}$$

where $C_s$ is the square of the original 'Smagorinsky constant'. The transfer through the cut (also called the subgrid drain or subgrid dissipation) is

$$\epsilon_s = 2\nu_s \bar{s}^2 = C_s h^2 (2\bar{s}^2)^{3/2} \ , \tag{4}$$

and hence

$$\bar{s}^2 = \epsilon_s^{2/3} (C_s h^2)^{-2/3}/2 \ , \tag{5}$$

which is related to an expression deriving directly from the Kolmogorov spectrum,

$$\bar{s}^2 = \int_0^{\pi/h} k'^2 E(k')dk' = \frac{3\alpha}{4}\epsilon^{2/3}(\pi/h)^{4/3} . \tag{6}$$

(The choice of $\pi/h$ as the cutoff wavenumber in this integral is necessarily arbitrary, since the value depends on the form of discretisation used. Lilly (1967) suggests it as the 'largest wavenumber unambiguously representable on a finite difference mesh'. Since changing the coefficient $\pi$ simply alters the Smagorinsky constant in a corresponding manner, we shall not discuss the question further.) The value of $\alpha$, on the other hand, is deduced from experiment, and might differ from flow to flow. The value used for illustrative purposes in all the figures in this paper is 55/36.

From the above we must have

$$C_s = \left(\frac{2}{3\alpha}\right)^{3/2} \pi^{-2} \tag{7}$$

This assumes that the transfer through the cut, $\epsilon_s$, can be identified with the total dissipation rate $\epsilon$. If viscous dissipation is non-negligible, it would be more accurate to identify the total dissipation occurring in the Kolmogorov spectrum as

$$\epsilon = \epsilon_r + \epsilon_s = 2(\nu + \nu_s)\bar{s}^2 , \tag{8}$$

where we distinguish the resolved dissipation $\epsilon_r$ and the subgrid drain $\epsilon_s$. Substituting for $\epsilon$ we get

$$s^3 = (3\alpha/4)^{3/2}(\pi/h)^2(\nu + \nu_s)2\bar{s}^2 , \tag{9}$$

giving

$$\nu_t = \nu + \nu_s = (4/3\alpha)^{3/2}(h/\pi)^2\bar{s}/2 = C_s h^2\sqrt{2\bar{s}^2} . \tag{10}$$

The Smagorinsky-Lilly (SL) model thus becomes a model for the total viscosity rather than the subgrid viscosity alone. The usual practice of adding the molecular viscosity to the subgrid viscosity computed from a formula such as (3) is clearly not consistent, since the model (3) is only valid if the viscous dissipation at the scale of $h$ is negligible compared with the subgrid drain, and hence if $\nu$ is negligible compared with $\nu_s$.

The modified SL model (10) would suggest that the subgrid viscosity should be zero wherever

$$\nu > (2/3\alpha)^{3/2}(h/\pi)^2\sqrt{2\bar{s}^2} \tag{11}$$

that is, when

$$h < (3\alpha/2)^{3/4}\pi(\nu^3/\epsilon_r)^{1/4} . \tag{12}$$

Thus, with this crude approach, the limit of direct simulation would be related simply to the ratio of the mesh resolution $h/\pi$ to a dissipation length scale $\eta_r$ derived from the resolved dissipation,

$$\eta_r = 1/k_r = (\nu^3/\epsilon_r)^{1/4} . \tag{13}$$

The above argument of Smagorinsky and Lilly is founded on the proportionality between the viscous dissipation of the resolved motions

$$\epsilon_r = 2\nu s^2 = 2\nu \int_0^{\pi/h} k'^2 E(k')dk' \tag{14}$$

and the subgrid drain

$$\epsilon_s = 2\nu_s s^2 = C_s h^2 (2s^2)^{3/2} , \tag{15}$$

or, with the suggested modification (10), with the total dissipation

$$\epsilon = 2\nu_t s^2 = C_s h^2 (2s^2)^{3/2} . \tag{16}$$

It is possible (and useful for the subsequent development of models using forms of the dissipation spectrum) to formulate the argument in these terms without reference to $s^2$ directly. We first introduce a number of non-dimensional variables,

$$\begin{aligned} \breve{\nu} &= \frac{\nu_s}{\nu} \\ \kappa &= k/k_d = k(\nu^3/\epsilon)^{1/4} \\ r &= \frac{h^2(2s^2)^{1/2}}{\nu} , \end{aligned} \tag{17}$$

where the dissipation wavenumber is

$$k_d = 1/\eta = (\epsilon/\nu^3)^{1/4} . \tag{18}$$

We can then write the resolved dissipation as

$$\begin{aligned} \epsilon_r(k) &= 2\nu \int_0^k k'^2 E(k')dk' \\ &= \frac{3\alpha\nu}{2}\epsilon^{2/3}k^{4/3} \\ &= \frac{3\alpha}{2}\epsilon\kappa^{4/3} , \end{aligned} \tag{19}$$

where k is the wavenumber of the cut, $(\pi/h)$. From(14) and (16) we know that

$$\frac{\epsilon_r}{\epsilon} = \frac{2\nu s^2}{2(\nu + \nu_s)s^2} = \frac{1}{1+\breve{\nu}} . \tag{20}$$

Thus we arrive at a non-dimensional relation of the form

$$\frac{1}{1+\breve{\nu}(\kappa)} = \frac{3\alpha}{2}\kappa^{4/3} . \tag{21}$$

That this leads to the model (10) is not immediately clear; we need to note that

$$\kappa = \frac{\pi}{h}\left(\frac{\nu^3}{\epsilon}\right)^{1/4} = \pi r^{-1/2}(1+\breve{\nu})^{-1/4} , \tag{22}$$

so that (21) becomes

$$(1+\breve{\nu})^{-1} = \frac{3\alpha}{2}\pi^{4/3}r^{-2/3}(1+\breve{\nu})^{-1/3} \; , \tag{23}$$

from which a low-Reynolds-number version of the SL model immediately follows in the non-dimensional form

$$\begin{aligned} 1+\breve{\nu} &= C_s r \qquad\qquad & r > C_s^{-1} \\ \breve{\nu} &= 0 & r < C_s^{-1} \end{aligned} \tag{24}$$

which is equivalent to (10). This model is usable at any resolution, but is an extremely crude modification based on a $k^{-5/3}$ spectrum that continues to the scale $r = C_s^{-1}$ and then vanishes, that is to the wavenumber

$$k_c = \left(\frac{2}{3\alpha}\right)^{3/4} k_r \tag{25}$$

for which

$$\int_0^{k_c} k^2 \alpha \epsilon^{2/3} k^{-5/3} dk = \epsilon \; . \tag{26}$$

Thus the model (24) is based on a Kolmogorov spectrum that is cut off at $k_c$:

$$\begin{aligned} E(k) &= \alpha\epsilon^{2/3}k^{-5/3} \qquad\qquad & k < k_c \\ E(k) &= 0 & k > k_c \; . \end{aligned} \tag{27}$$

The original SL model (3) is equivalent to

$$\breve{\nu} = C_s r \; . \tag{28}$$

The subscript $s$ is appended to the constant to denote that its value (7) has been derived using the arguments of Smagorinsky and Lilly from the Kolmogorov inertial-range spectrum.

$C_s$ is a constant. In general, using other forms of spectrum designed to describe the dissipation range of wavenumbers, relations analogous to (24) can be obtained of the general form $r = r(\breve{\nu})$, though we shall find that some cannot be rewritten in the form $\breve{\nu} = \breve{\nu}(r)$ explicitly. The remainder of the paper shows how some of these functions may be extracted and utilised.

## 3 Dissipation Range Models

The cut off Kolmogorov spectrum (27) is a crude and clearly unphysical representation of what happens in the dissipation range of wavenumbers $k \sim k_d$. In reality we know that the spectrum must fall smoothly below the $k^{-5/3}$ behaviour, in such a way that the constraint

$$2\nu \int_0^\infty k^2 E(k) dk = \epsilon \tag{29}$$

is fulfilled. The generic form, based on the Kolmogorov (1941) spectrum, is

$$E(k) = \alpha \epsilon^{2/3} k^{-5/3} f(\kappa) \tag{30}$$

A number of authorities have given forms of the spectrum with these properties, and since none is without its supporters and its critics we a shall derive dissipation-range subgrid-scale models for several of the best known of these spectra.

Shortly after the work of Kolmogorov, Obukhov (1941) suggested a form of the spectrum for the dissipation range based on physical arguments. His spectrum has a number of deficiencies, notably that, like the Kolmogorov spectrum, its integral is infinite, and the spectrum has to be cut off at a finite wavenumber to satisfy the constraint (29). For this and other reasons we shall not consider Obukhov's spectrum further.

Heisenberg (1948), employing an assumption that will be familiar to all practicioners of large-eddy simulation that the wavenumbers higher than $k$ act on those lower than $k$ by an eddy viscosity, derived an integral relation for the spectrum through the dissipation range. Chandrasekhar (1949) showed that the solution was

$$E(k) = \alpha \epsilon^{2/3} k^{-5/3} \left[ 1 + \left( \frac{3\alpha}{2} \right)^3 \kappa^4 \right]^{-4/3} \tag{31}$$

This spectrum satisfies the contraint (29), and tends to the Kolmogorov spectrum for small $\kappa$. At large $\kappa$ it has the less desirable property of an asymptotic power-law behaviour

$$E(k) \sim k^{-7} . \tag{32}$$

Kovasznay (1948) proposed an even simpler dissipation-range spectrum based on an assumed algebraic relation between the spectral transfer and the energy spectrum, resulting in the form

$$E(k) = \alpha \epsilon^{2/3} k^{-5/3} \left( 1 - \frac{\alpha}{2} \kappa^{4/3} \right)^2 . \tag{33}$$

This also satisfies (29) and has the $k^{-5/3}$ spectrum as a small $\kappa$ limit, but suffers from the defect that the spectrum vanishes at

$$\kappa = (2/\alpha)^{3/4} \,, \tag{34}$$

at which point the integral (29) is already equal to $\epsilon$ and so the energy must be zero for all higher wavenumbers. Nevertheless credible and instructive models can be constructed from this spectrum.

Pao (1965) suggested a simple and attractive hypothesis, also on the basis of an algebraic relation between the transfer and energy spectra. His spectrum,

$$E(k) = \alpha\epsilon^{2/3}k^{-5/3}\exp\left(-\frac{3\alpha}{2}\,\kappa^{4/3}\right) \,, \tag{35}$$

satisfies (29) and matches to the Kolmogorov spectrum at low wavenumbers, but it also has a healthy high wavenumber behaviour.

Other dissipation spectra have been suggested, for instance by Townsend (1951) who used a physical hypothesis to arrive at the form

$$E(k) = \sqrt{2/\pi}\,\epsilon^{2/3}k^{-5/3}\,\kappa^{-1/3}\exp(-2\kappa\,) \,. \tag{36}$$

A spectrum such as this is not of great utility for the construction of a subgrid-scale model, however, since it is intended purely as a description of the high wavenumber dissipative range, and does not tend to the inertial-range spectrum at the low $k$ end. As a result, any model based on this spectrum will not have the SL model as a coarse resolution limit. We shall restrict our attention to those spectra that do satisfy this important limit, as well as the constraint (29). Figure 1 shows the spectra in a familiar manner, and Figure 2 shows $k^2E(k)$ in the dissipation range.

We now turn to the forms of $\epsilon_r/\epsilon$ for each of the spectra that have the desired properties. Each of the three resulting integrals

$$\epsilon_r(k) = 2\nu\int_0^k k'^2E(k')\,dk' \tag{37}$$

is tractable. The Heisenberg-Chandrasekhar (HC) spectrum gives

$$\frac{\epsilon_r}{\epsilon} = \kappa^{4/3}\left[\left(\frac{2}{3\alpha}\right)^3 + \kappa^4\right]^{-1/3} \,. \tag{38}$$

Kovasznay's spectrum gives

$$\frac{\epsilon_r}{\epsilon} = 1 - \left(1 - \frac{\alpha}{2}\,\kappa^{4/3}\right)^3 \,, \tag{39}$$

while Pao's gives

$$\frac{\epsilon_r}{\epsilon} = 1 - \exp\left(-\frac{3\alpha}{2}\,\kappa^{4/3}\right) \ . \tag{40}$$

Before proceeding, we note briefly that while none of the above expressions looks very similar to (21) superficially, the power series expansions in $\kappa$ (for small $\kappa$) do turn out each to have a leading term $\kappa^{4/3}$, which is necessarily the case if the resulting model is to be compatible with the SL form at low wavenumbers.

Each of the preceding expressions is now equated to $1/(1+\breve{\nu})$, we substitute (22) for $\kappa$, and extract $r$. The results are, for HC

$$r = C_s^{-1}\,(1+\breve{\nu})^{-1/2}\,[\,(1+\breve{\nu})^3 - 1\,]^{1/2}\,, \tag{41}$$

for Kovasznay

$$r = 3^{-3/2} C_s^{-1}\,(1+\breve{\nu})^{-1/2}\,\left[1 - \left(\frac{\breve{\nu}}{(1+\breve{\nu})}\right)^{1/3}\right]^{-3/2}\,, \tag{42}$$

and for Pao

$$r = C_s^{-1}\,(1+\breve{\nu})^{-1/2}\,\left[\log\left(\frac{1+\breve{\nu}}{\breve{\nu}}\right)\right]^{-3/2}\,. \tag{43}$$

These expressions are directly comparable to (24) and (28). The compatibility between the expressions in the limit of large $\breve{\nu}$ and $r$, which corresponds to low $\kappa$, is only transparent for (41). Only one of the relations can be algebraically inverted, that from the Kovasznay spectrum, giving

$$\breve{\nu} = \frac{1}{8}\left(a + \sqrt{\frac{4-a^3}{3a}}\right)^3 - 1\,, \tag{44}$$

where

$$a = \frac{\alpha}{2}\left(\frac{\pi^2}{r}\right)^{2/3}\,, \tag{45}$$

which clearly does have (28) as a high $r$ asymptote. Another strategy is to expand the functions $r(\breve{\nu})$ as power series in $\breve{\nu}$ about $\breve{\nu} = \infty$. We obtain, for HC

$$r = C_s^{-1}\left(\breve{\nu} + 1 - \frac{1}{2\breve{\nu}^2} + \frac{1}{\breve{\nu}^3} - \frac{3}{2\breve{\nu}^4} + \ldots\right)\,, \tag{46}$$

for Kovasznay

$$r = C_s^{-1}\left(\breve{\nu} + \frac{1}{2} - \frac{5}{72\breve{\nu}} + \frac{5}{144\breve{\nu}^2} + \ldots\right)\,, \tag{47}$$

and for Pao

$$r = C_s^{-1}\left(\breve{\nu} + \frac{1}{4} - \frac{1}{32\breve{\nu}} + \frac{1}{128\breve{\nu}^2} + \ldots\right)\,. \tag{48}$$

These expressions are most revealing. While the recovery of the original SL model for the limit $\breve{\nu} \to \infty$ in each case is reassuring, we note that each of the new models has the SL model as a parallel asymptote in the high $\breve{\nu}$ (and high $r$) limit, with an additive offset $\beta$:

$$\breve{\nu} \to C_s r - \beta \, . \tag{49}$$

For HC, $\beta = 1$, and the model is asymptotic to the modified low-Reynolds-number SL model (24). The other two models have different asymptotes, though both are displaced from the SL line in the limit. The relations (41), (42) and (43) are shown in Figure 3, along with the lines corresponding to the original and modified SL models, (28) and (24). We note the displacements for large $r$, but also the fact that for the model based on the Kovasznay spectrum $\breve{\nu}$ vanishes at a finite $r$, which from (42) is at

$$r = \left(\frac{\alpha}{2}\right)^{3/2} \pi^2 = C_s^{-1} 3^{-3/2} \, . \tag{50}$$

If we require that the subgrid viscosity vanish at $r = 0$, it is possible to subtract this offset from $r$ to form a modified (shifted) model based on the Kovasnay spectrum but using in place of $r$

$$r - C_s^{-1} 3^{-3/2} \, . \tag{51}$$

The offset then becomes $\beta = 1/2 - 3^{-3/2} = 0.31$. The behaviour of this model turns out to be very close to that based on the Pao spectrum for which $\beta = 1/4$ and which is our favoured model for practical use.

# 4 Practical Application

In a real simulation, it is important that the subgrid-scale eddy viscosity should be quickly computable as well as physically realistic. Functions of the form $r = r(\breve{\nu})$ are impractical since they give $\breve{\nu}$ implicitly. The Kovasznay form is attractive since it can be inverted (44), but in practice it is easy to find simple functions that fit the desired behaviour of $\breve{\nu}(r)$ rather accurately. These functions can be cast in terms of the original SL eddy viscosity $\breve{\nu}_{sl} = r/C_s$, and they take the form

$$\breve{\nu}_s = \breve{\nu}_{sl} - \beta[1 - \exp(-\breve{\nu}_{sl}/\beta)] \, , \tag{52}$$

which clearly has the parallel asymptote $\breve{\nu} \to \breve{\nu}_{sl} - \beta$ and $\breve{\nu} = 0$ when $r = 0$. Such expressions are simple to compute and can be programmed rapidly as modifications to an existing SL model algorithm:

$$\nu_s = \nu_{sl} - \beta\nu[1 - \exp(-\nu_{sl}/\beta\nu)] \, . \tag{53}$$

Figure 4 shows a comparison of fitting functions (52) with $\beta = 0.25$ and 0.31, the P-model (Pao spectrum), and the shifted K-model (Kovasznay spectrum, modified using (51)). The close correspondence between these functions makes it difficult to distinguish between them graphically. Our preferred function for practical computations is the fit (52) with $\beta = 2/9$, originally put forward on the basis of heuristic arguments that parallel those give here by Voke (1991).

Figures 5 to 8 show the mean velocity and r.m.s. turbulence intensities from two of a set of low-Reynolds-number channel flow LES performed by Zhao (1994) at modest resolution ($96 \times 80 \times 64$, in a box of dimension $4\pi\delta \times 2\pi\delta \times 2\delta$, or $2576 \times 1288 \times 410$ in wall units). Zhao (1994), like Yang and Voke (1993), uses a subgrid-scale model very similar to the classical Smagorinsky-Lilly model but responding only to the fluctuating strain rate, not to the mean strain, and with wall damping functions. Predictions obtained with this model are compared with those using the model modified using (53) with $\beta = 2/9$, and with PIV experimental data provided by Kasagi (Nishino and Kasagi 1989) for flow in a water-channel at the identical Reynolds number, $\delta^+ = 205$. It is clear that the low-Reynolds-number subgrid-scale model is generally beneficial to the performance of the LES, and clearly illustrates the effects of the over-estimation of the eddy viscosity present in the classical model.

# 5 Conclusions

It has been possible to derive subgrid-scale models, analogous to the Smagorinsky-Lilly model, from several of the best-known forms for the turbulence energy spectrum in the dissipation range. These models are both interesting and credible, reducing the eddy viscosity greatly when the mesh cutoff approaches the dissipation length scale and demonstrating a constant offset, in terms of the molecular viscosity, in the inertial range, whose magnitude depends on the model chosen. Practical computation methods, based on simple functions that fit the forms of the exact models closely, have also been given, and the beneficial effect of the use of such a low-Reynolds-number model in a real large-eddy simulation has been demonstrated briefly.


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# Figure Captions

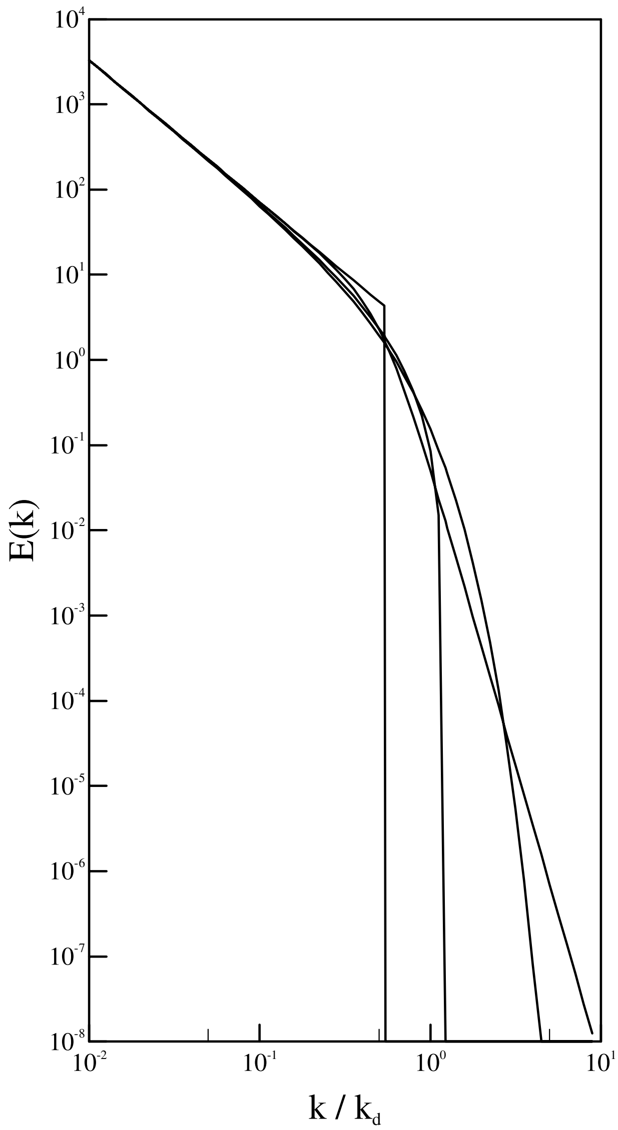


Figure 1. Hypothetical energy spectra in the inertial and dissipation ranges. Left to right at the bottom: cut off Kolmogorov spectrum; Kovasznay (1948); Pao (1965); Heisenberg (1948).

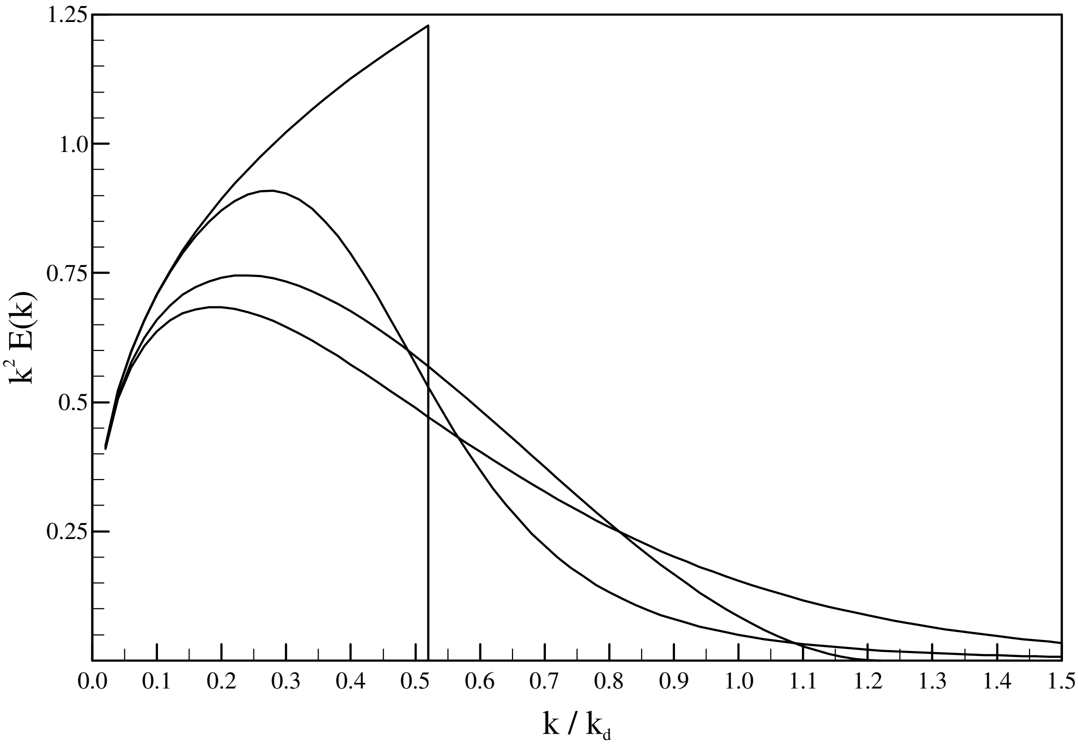


Figure 2. Hypothetical dissipation spectra $k^2E(k)$ in the dissipation range. Top to bottom on the left: cut off Kolmogorov spectrum; Heisenberg (1948); Kovasznay(1948); Pao (1965).

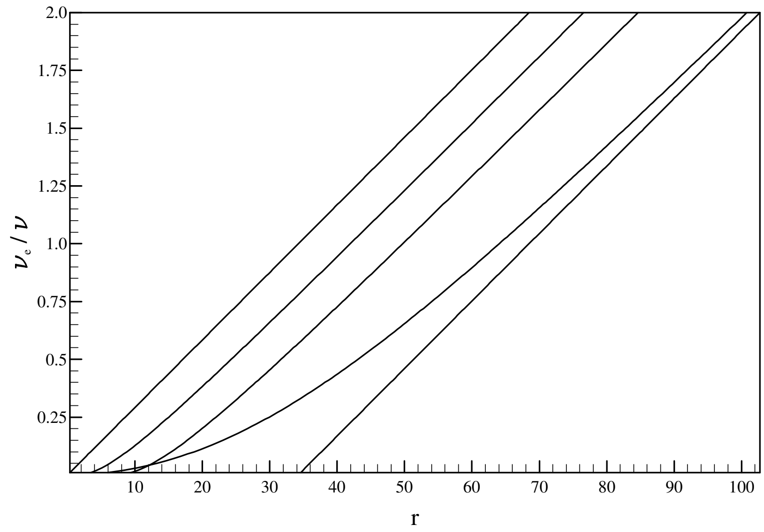


Figure 3. Behavior of the nondimensional subgrid-scale eddy viscosity $\breve{\nu}$ as a function of $r$. Top left to bottom right: Smagorinsky-Lilly model; model based on Pao (1965) spectrum; based on Kovasznay (1948) spectrum; based on Heisenberg-Chandrasekhar spectrum; based on cut off Kolmogorov spectrum.

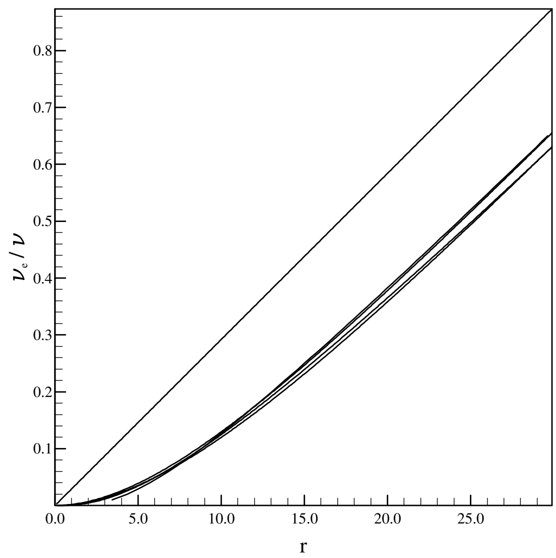


Figure 4. Behaviour of the selected subgrid-scale eddy viscosities $\breve{\nu}$ as a function of $r$ in the dissipation range. Diagonal line, Smagorinsky-Lilly; other lines, left to right, model based on Pao (1965) spectrum; fit (53) with $\beta = 0.25$; shifted model based on Kovasznay (1948) spectrum; fit (53) with $\beta = 0.3075$.

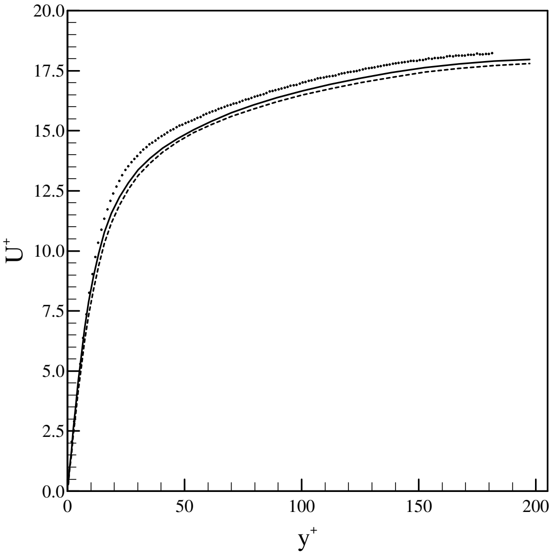


Figure 5. Comparison of mean velocity profiles for a turbulent channel flow at $h^+ = 205$, from Zhao (1994): dashed line, standard Smagorinsky-Lilly model using fluctuating strain rate only; solid line low-Reynolds-number model; points, experiment of Nishino and Kasagi (1989).

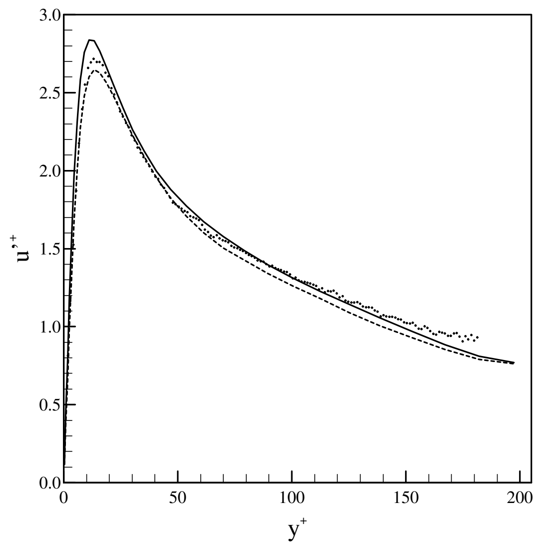


Figure 6. Comparison of r.m.s. intensity $u'$ profiles for a turbulent channel flow at $h^+ = 205$, from Zhao (1994): as Figure 5.

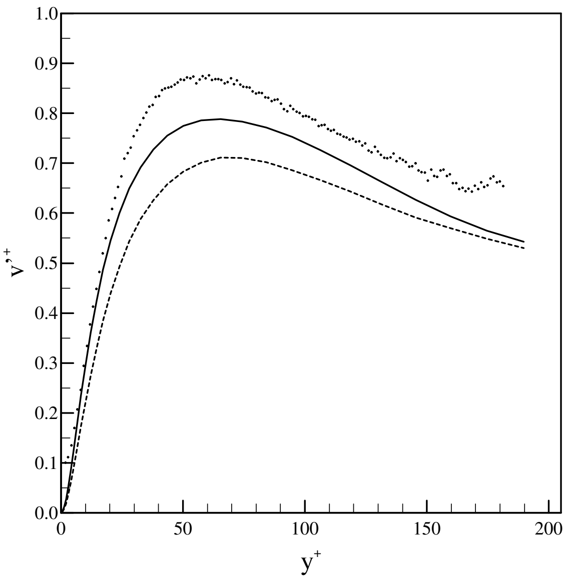


Figure 7. Comparison of r.m.s. intensity $v'$ profiles for a turbulent channel flow at $h^+ = 205$, from Zhao (1994): as Figure 5.

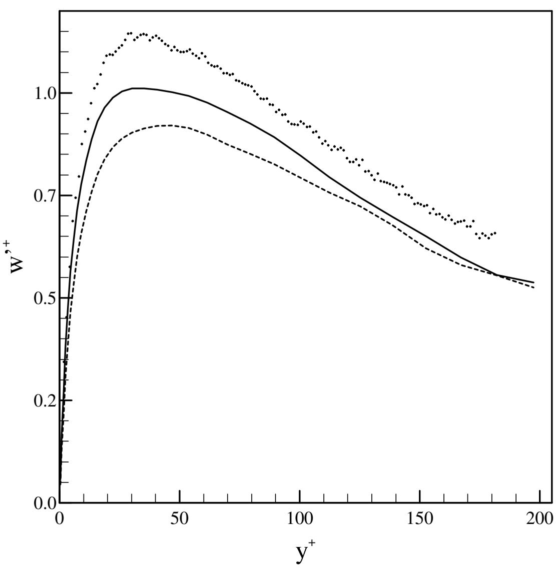


Figure 8. Comparison of r.m.s. intensity $w'$ profiles for a turbulent channel flow at $h^+ = 205$, from Zhao (1994): as Figure 5.