Absorbing Re-reflected Random Waves in SPH

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SPHERIC workshop
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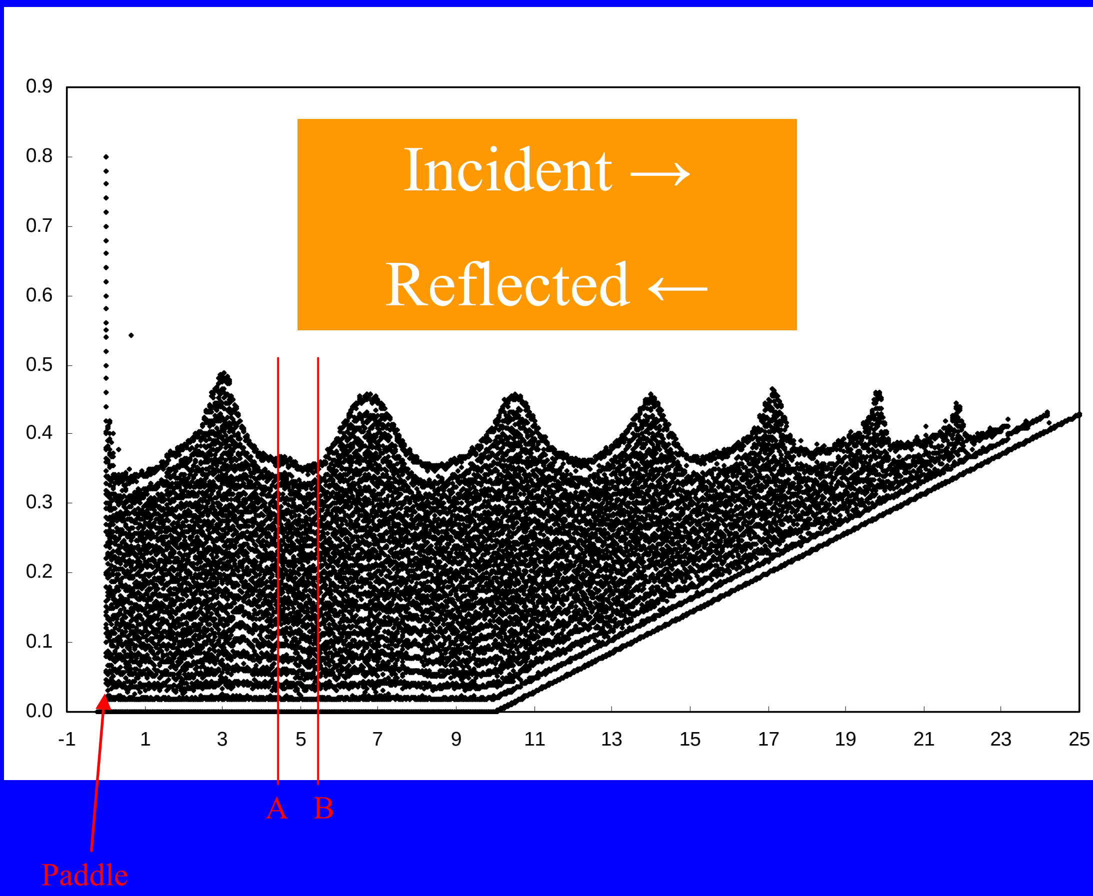
Outline

- **Motivation**
- **Re-reflection from Wave paddle**
  - Frigaard & Brorsen model
- **Implementation in SPH**
- **Conclusions and Further Work**

Motivation

- *Wave overtopping*
  - structures can be highly reflective
  - limited info from regular waves
- *Physical / numerical modelling*
  - need random waves
  - reflected waves re-reflected by paddle
    - can spoil statistics of random wave train
- *Preventing re-reflection*
  - need to know about reflected waves at paddle
  - method of Frigaard & Brorsen
- *How does method work in SPH?*
  - Noisy data
    - need to 'clean' data
  - What are issues?

Frigaard-Brorsen Method



- ***Outline of method***
  - *Paddle at x_0,*
  - *Measurements at A (x_1), B (x_2)*

$$\eta(x,t) = \eta_I(x,t) + \eta_R(x,t)$$
$$= a_I \cos(2\pi ft - kx + \Phi_I) + a_R \cos(2\pi ft + kx + \Phi_R)$$

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Frigaard-Brorsen Method

$$\eta^*(x_n,t)$$
$$= Ca_I \cos(2\pi ft - kx_n + \Phi_I + \Phi_n{}^{th})$$
$$+ Ca_R \cos(2\pi ft + kx_n + \Phi_R + \Phi_n{}^{th})$$

$$\eta^{calc}(t) = \eta^*(x_1,t) + \eta^*(x_2,t)$$

$$\eta^{calc}(t) = \eta_R(x_0,t) \quad when$$
$$\Phi_2^{th} = -k(x_1 - x_0) + \pi/2$$
$$\Phi_1^{th} = -k(x_2 - x_0) - \pi/2$$
$$C = 1/[2\cos(-k(x_2 - x_1) - \pi/2)]$$

– *In frequency space, amplify and phase shift signals from measurement stations 1, 2 then combine together*

Frigaard-Brorsen Method

- *Process (conceptual)*
  - *Take (Discrete) Fourier transform of signals*
    - Y_1, Y_2
  - *Multiply by digital filters with impulse responses H_1, H_2 to amplify and phase-shift signals*
    - $H_1 Y_1, H_2 Y_2$
  - *combine together*
    - $H_1 Y_1 + H_2 Y_2$
  - *Inverse (Discrete) Fourier transform gives required signal*
    - *inverse of product of DFT's is discrete convolution*

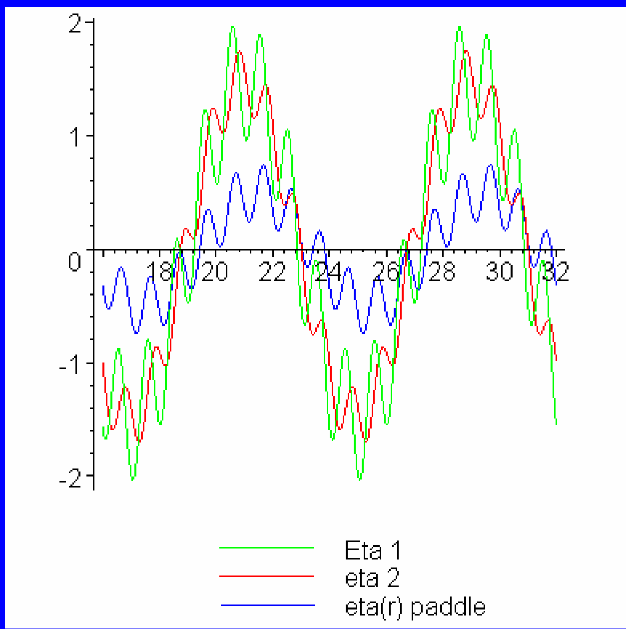
Frigaard-Brorsen Method

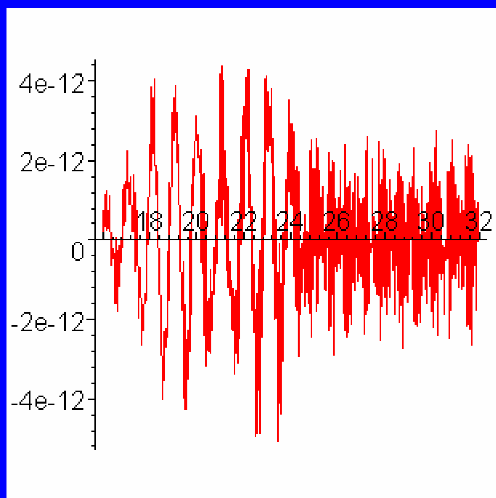
- ***Process (in practice):***
  - *Find inverse (Discrete) Fourier transform of H_1, H_2 giving h_1, h_2*
  - *find discrete convolution of η_1 with h_1, η_2 with h_2 and add together to give reflected wave at paddle*

- **MAPLE code**

```
• for j from 1 to N do:
•   if (j <= NA) then:
•    pj:= p-j+1:
•   else:
•    pj:= p-j + N/2 +1:
•   end if:
• end if:
  deta:=h1[j]*eta1[pj]+h2[j]*eta2[pj]:
•   eta[p]:=eta[p]+deta:
• end do:
```

Algorithm test 1
f1 = df; f2 = 8df





Method is essentially exactly when frequency is integer multiple of df

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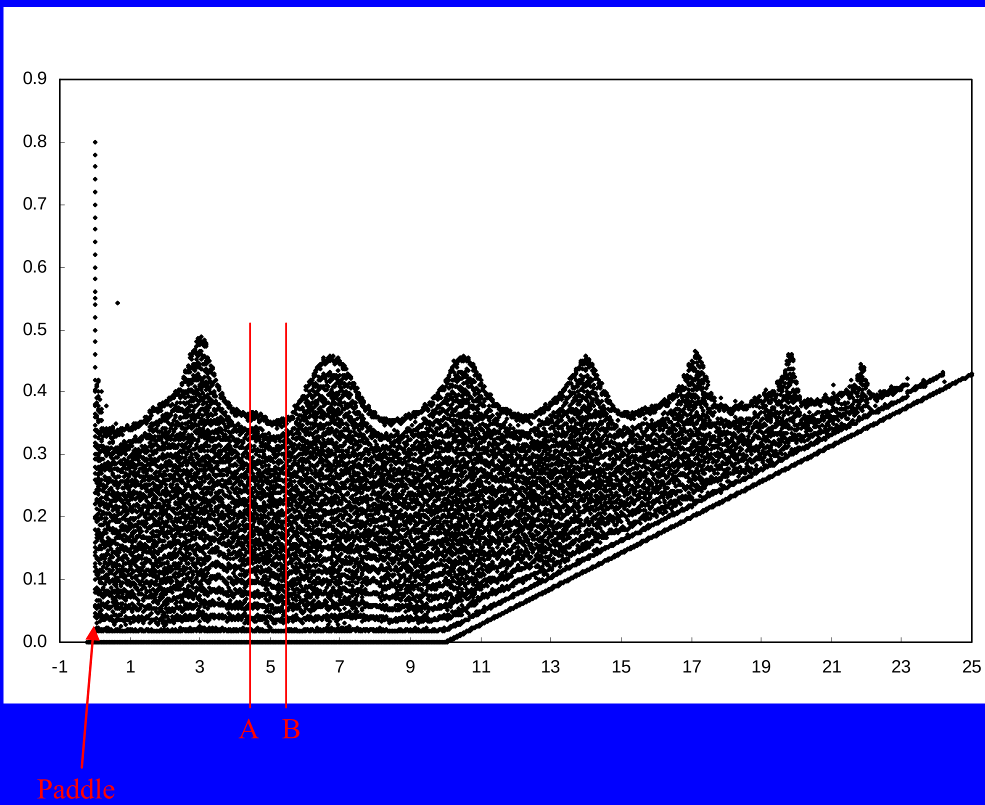
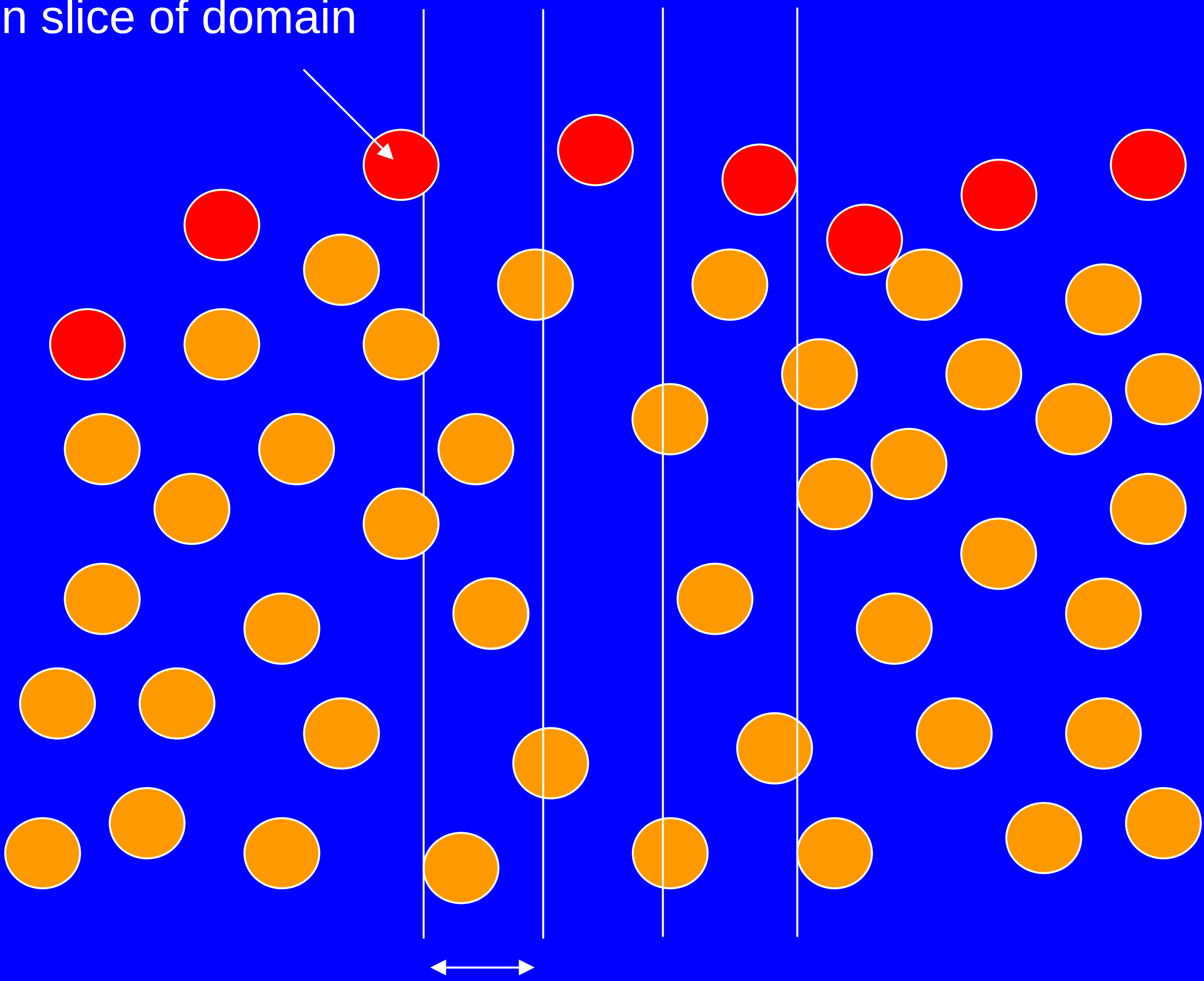
Implementation in SPH



- ***I-SPH: Songdong Shao***
- ***Details***
  - *dt = 2^(-10)~.001s*
  - *Depth 0.4m*
  - *Period 2s*
  - *Amplitude ~ 0.1m*
  - *18,800 particles*

Surface elevation in SPH

Free surface: highest
point in slice of domain



Slice width ~ 2h

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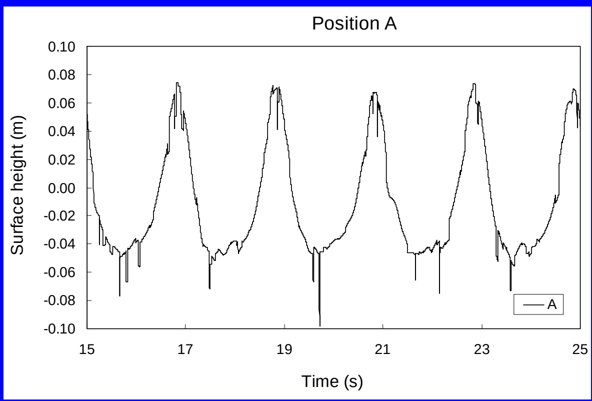
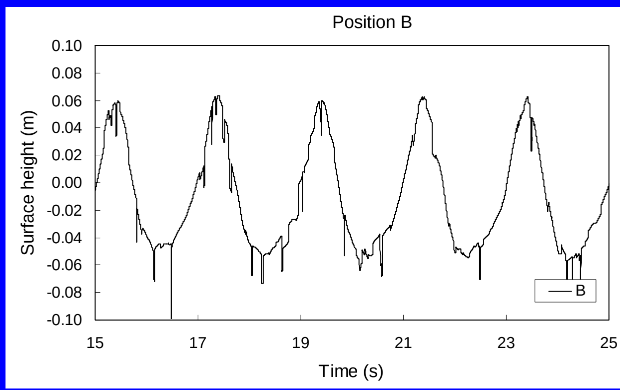
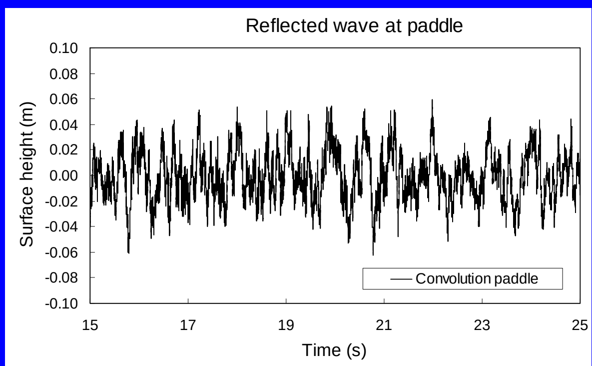
Implementation in SPH
df = .25; f = .5

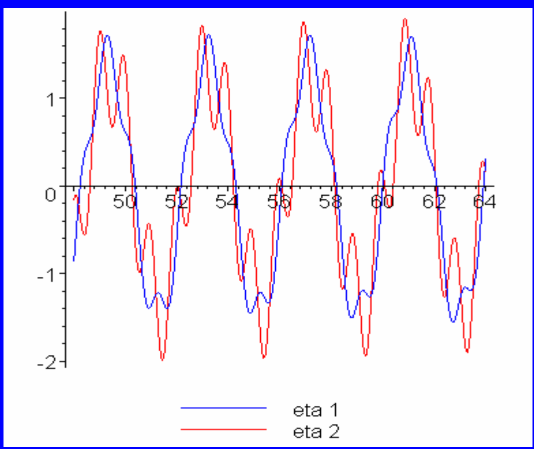


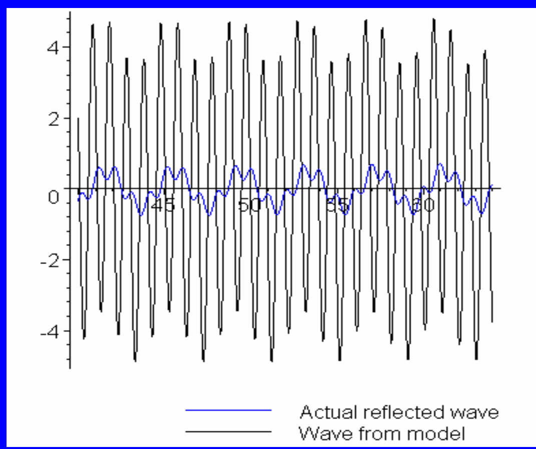




Noisy SPH data produce noisy output even when frequency is exact multiple of df!

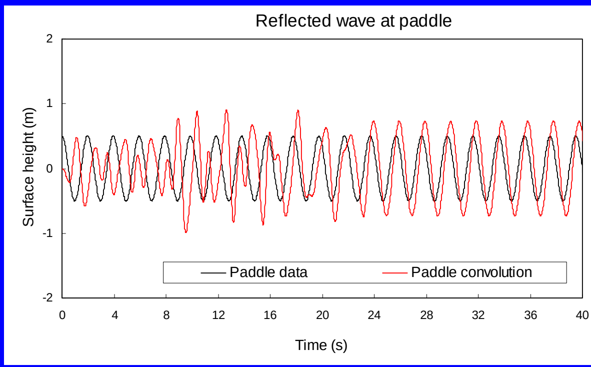
Algorithm test 2
f1 = 2.05df; f2 = 8.1df

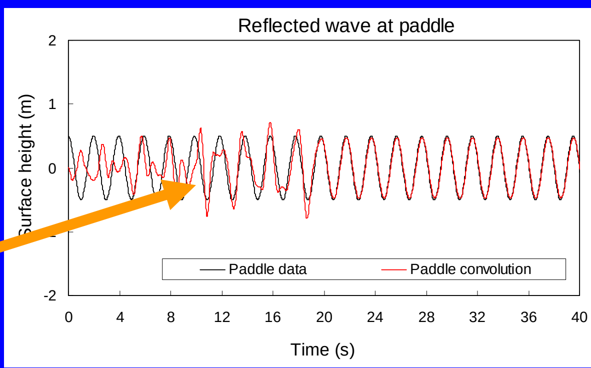




F-B Method performance degenerates when frequency is non-integer multiple of df.

Algorithm test 3
f1 = 4.05df





Filter needs to 'warm up'

'Circular' convolution (bottom figure) can improve performance but not available in real-time simulations

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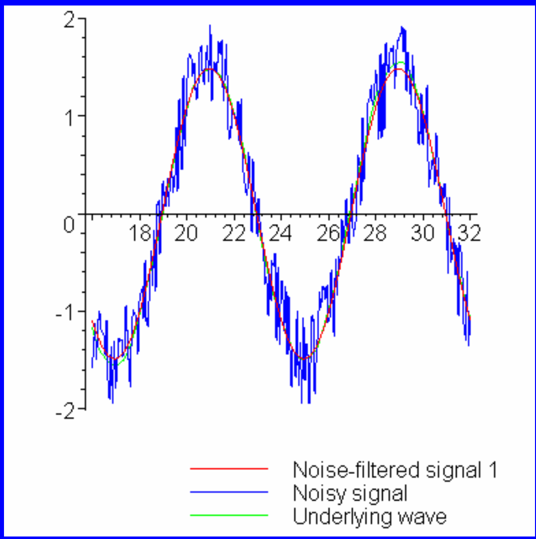
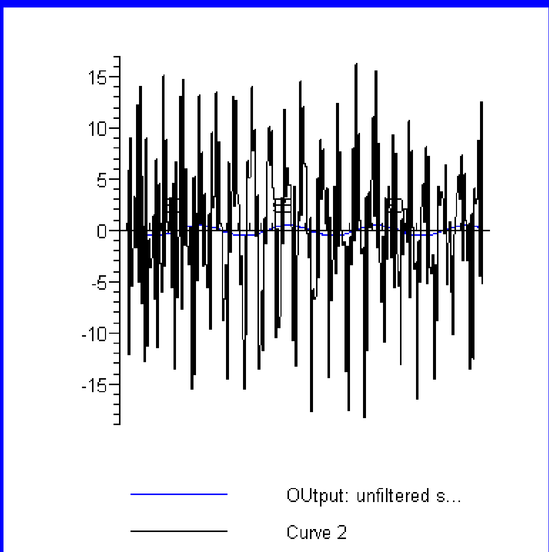
Algorithm test 4
Random noise





Frigaard-Brorsen does not work if 'noise' is random

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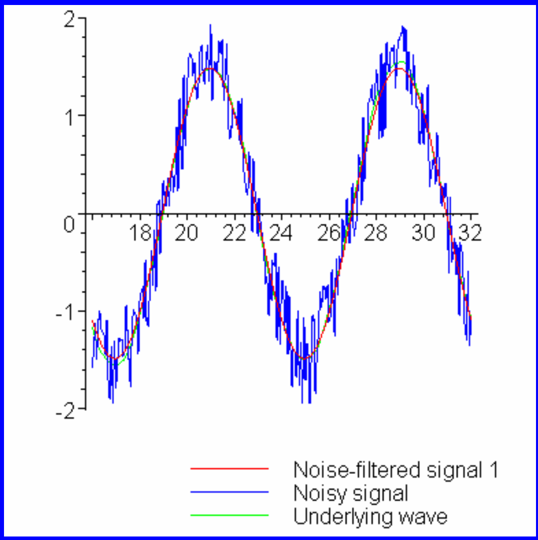
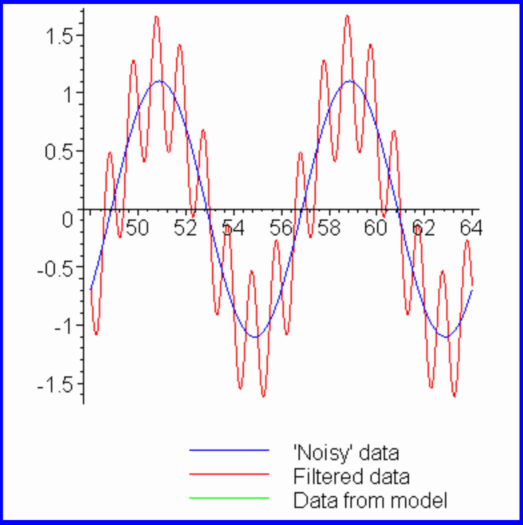
Noise filtering

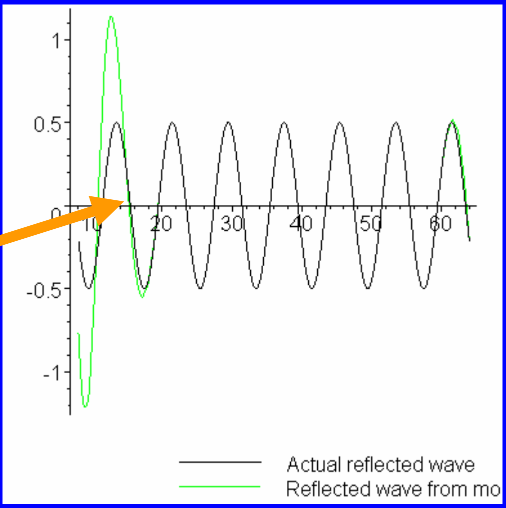


- ***Theory:***
  - *Signal consists of high-frequency 'noise' over desired low-frequency signal*

- ***Process:***
  - *Define low-pass digital filter f*
    - *frequency increment df*
    - *Cutoff frequency nmax * df*
  - *find discrete convolution of η_1, η_2 with f to give 'cleaned signal'*

Noise filtering test 1
f1=df; f2 =8df; cutoff 4df

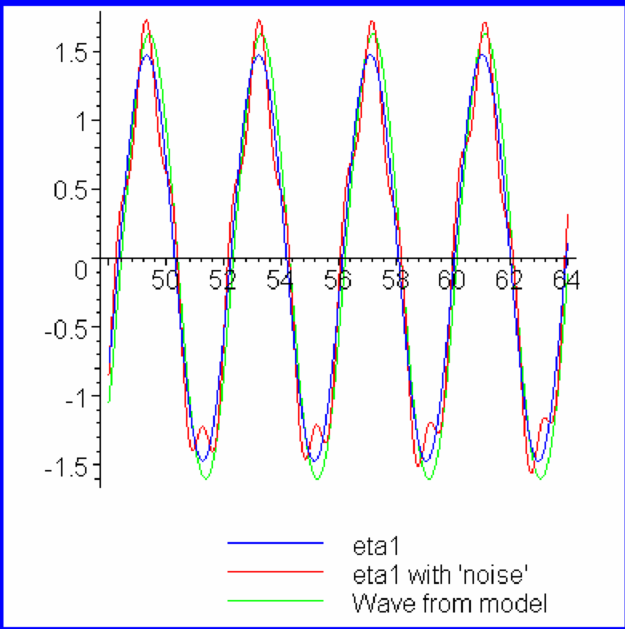


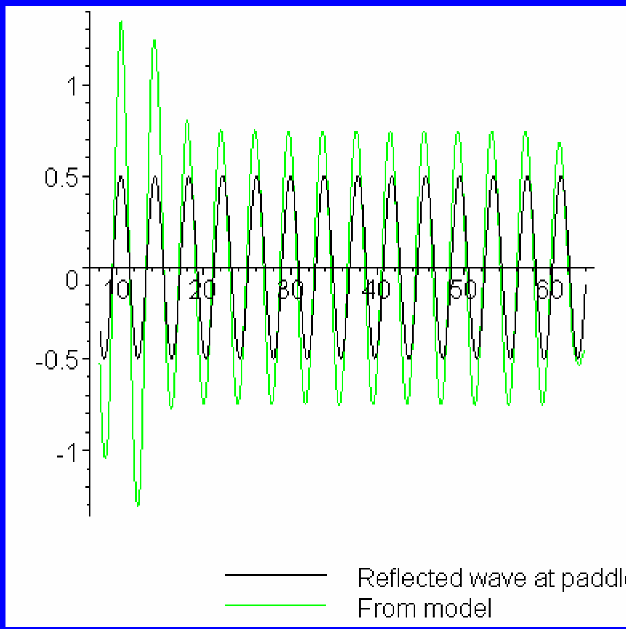


Filter needs to ‘warm up’

Noise filtering works in conjunction with Frigaard-Brorsen method if ‘noise’ is high-frequency sinusoid.

Noise filtering test 2
f1=2.01df; f2=8.1df; cutoff 4df





Noise filtering with Frigaard-Brorsen performance affected if frequencies aren't integer multiples of df

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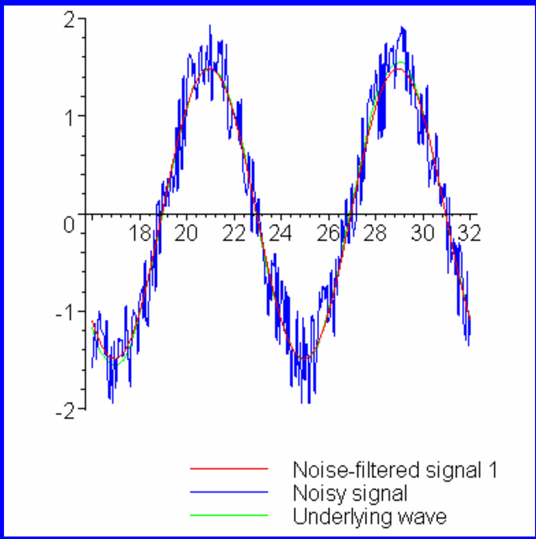
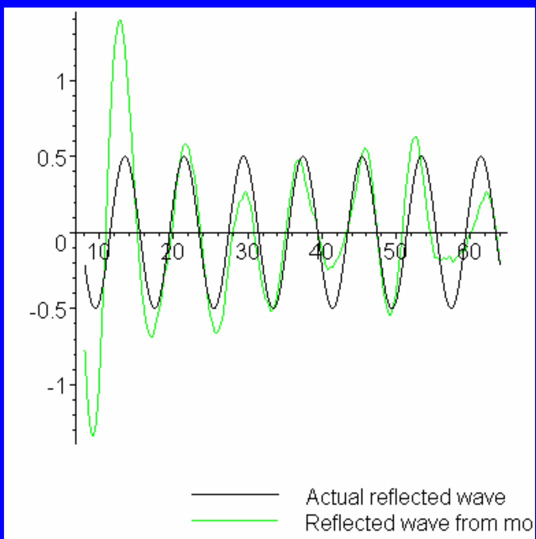
Noise filtering test 3
Random noise





Noise filtering with F-B works if 'noise' is random (but only with 'circular convolution' and then not as well as if noise is periodic)

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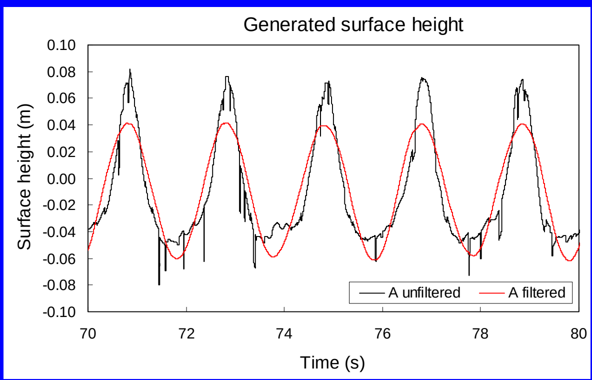
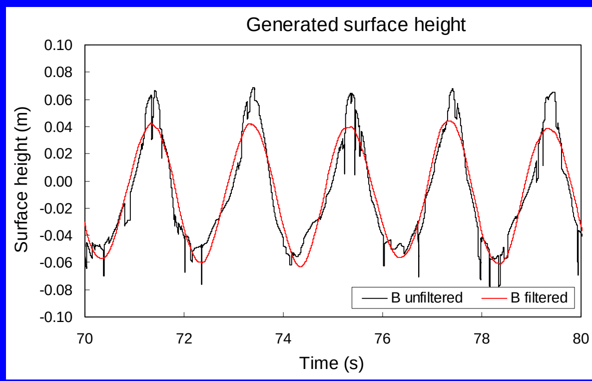
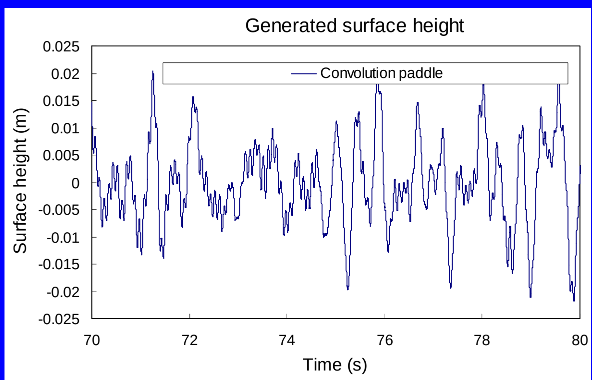
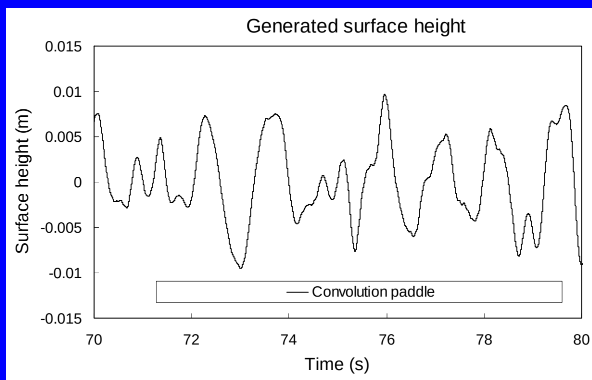
Implementation in SPH
df = .25; Wave f =.5,f<1.25,10^12 modes noise filter / convolution









Noise reduction ‘cleans’ SPH elevation signal: apparently effective

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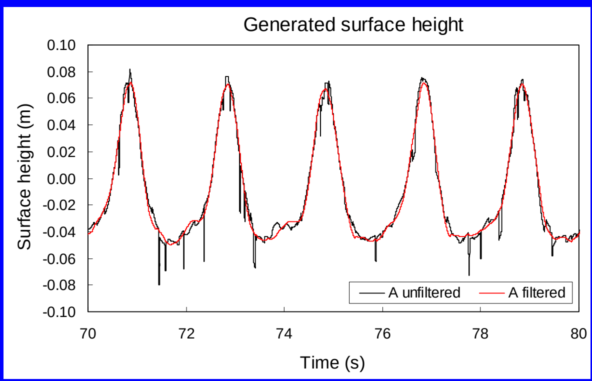
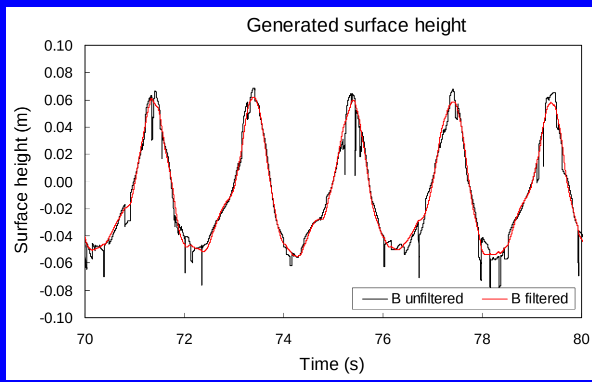
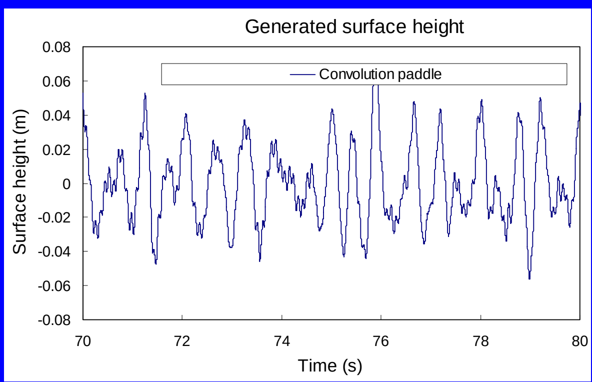
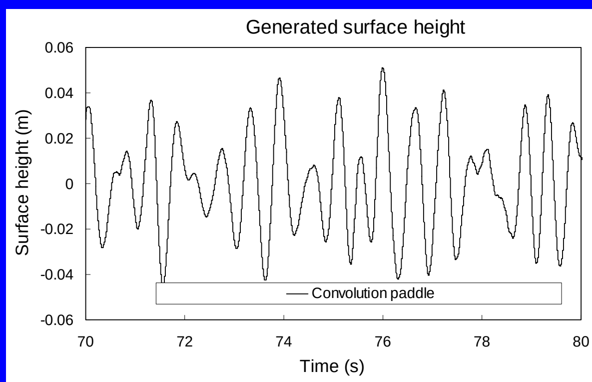
Implementation in SPH
df = .25; Wave f =.5,f<2.25,10^12 modes noise filter / convolution









But increasing resolution of wave increases computed magnitude of reflected wave????

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Development for Random Waves

- **F-B method**
  - Works with any combination of waves with frequency i*df
- **Spectrum**
  - Discrete approximation to given spectrum
  - Pseudo-random (actually periodic)
  - Components at integer multiples of df from noise filter / anti-reflection convolution
- **Implementation in SPH**
  - No further complications envisaged in addition to those from
    - Warm-up times for filters
    - Noisy data
    - Drift of frequencies from i*df

Conclusions and Further Work

- **Anti-reflection**
  - Frigaard-Brorsen method works with ideal examples
  - Deteriorates when frequencies depart from ideal
  - Works poorly with signals with random noise
  - 'circular convolution' can improve results but requires data from future
- **Implementation in SPH**
  - Noisy signals cause problems
  - Departure from desired frequency
  - 'circular convolution' not available for real-time SPH
- **Future**
  - Combine anti-reflection / noise reduction?
  - Better method to define surface height?
  - Translate reflected wave height to paddle motion
  - moving paddle

References

- **Peter Frigaard & Michael Brorsen (1995):** A time-domain method for separating incdent and reflected irregular waves. **Coastal Eng 24, 205-215.**

- **Songdong Shao and Edmond Y. M. Lo (2003):** Incompressible SPH method for simulating Newtonian and non-Newtonian flows with a free surface. **Journal of Advances in Water Resources,**

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