Presentation SPH workshop Rome

Two 2D SPH simulations

Lynyrd de Wit
wit@svasek.com
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Svasek Hydraulics (www.svasek.com)

Technische Universiteit Delft

- **Introduction**
- **SPH specifications**
- **Application of SPH**
  - Sharp weir
  - Waves on a beach
- **Conclusions**

- **MSc. Graduation at TU Delft, Civil Engineering about the possibilities of SPH in hydraulic engineering**
- **SPH was not used at the TU Delft before**
- **During graduation applied SPH 2D for**
  - Viscosity benchmarks: Poiseuille, Couette, Shear-driven cavity flow, to check artificial viscosity
  - Free surface problems: Breaking dam, bore at a wall, standing wave, waves on a beach, and a sharp weir
- **Currently working at Svasek Hydraulics, a specialized hydraulic consultant**
- **First time I meet other SPH users, I hope for interesting conversations**

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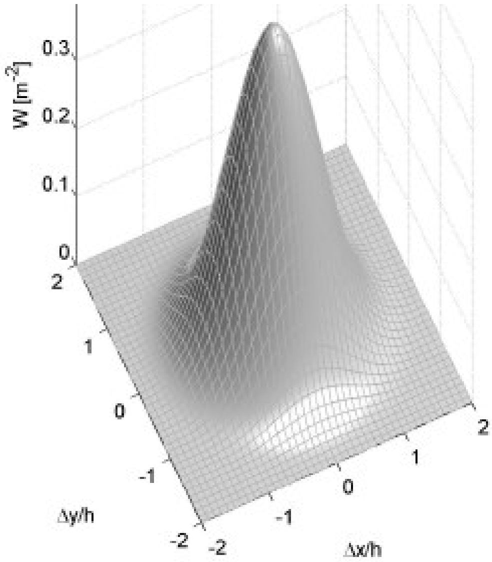
- **Piecewise cubic spline kernel**

$$W_{ij} = \frac{10}{7\pi h^2}\begin{cases} 1-\frac{3}{2}q^2+\frac{3}{4}q^3 & 0 \le q \le 1 \\ \frac{1}{4}(2-q)^3 & 1 \le q \le 2 \\ 0 & q > 2 \end{cases}$$

$$q = \frac{r}{h}; \;\; r = |\mathbf{r}_i - \mathbf{r}_j|; \quad h = 1.4\, r_{ini} \mapsto 20 \text{ neighbours}$$



- **Equations of motion (Monaghan 1994)**

$$\frac{D\rho_i}{Dt} = \sum_j m_j (\mathbf{u}_i - \mathbf{u}_j)\nabla_i W_{ij}$$

$$\frac{D\mathbf{u}_i}{Dt} = -\sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij} + \mathbf{f}_i$$

$$\frac{D\mathbf{x}_i}{Dt} = \mathbf{u}_i + 0.5 \sum_j m_j \left(\frac{\mathbf{u}_j - \mathbf{u}_i}{\bar{\rho}_{ij}} \right) W_{ij} \quad \text{(XSPH)}$$

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SPH specifications

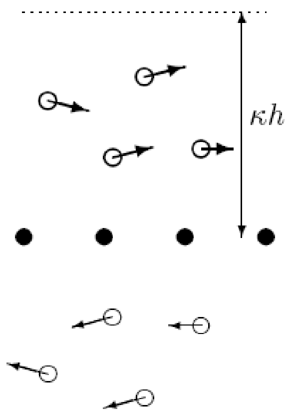
- **Equation of state (Batchelor 1974)**

$$p_i = \frac{c^2 \rho_0}{7}\left[\left(\frac{\rho_i}{\rho_0}\right)^7 - 1\right] \qquad c = 10 u_{\max} \qquad \mapsto \qquad \frac{\Delta\rho}{\rho} \approx 0.01$$

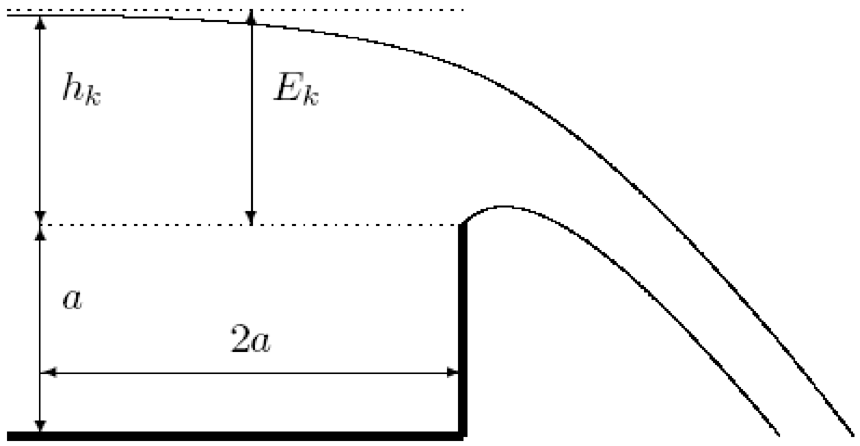
- **Artificial viscosity (Monaghan 1994)**

$$\Pi_{ij} = \begin{cases} \dfrac{-\alpha h c}{\overline{\rho}_{ij}} \dfrac{\mathbf{u}_{ij}\cdot\mathbf{r}_{ij}}{r_{ij}^2+\varphi^2} & \mathbf{u}_{ij}\cdot\mathbf{r}_{ij} < 0 \\ 0 & \mathbf{u}_{ij}\cdot\mathbf{r}_{ij} \geq 0 \end{cases}$$

- **Closed boundaries**
  - Boundary particles with Leonard Jones force (Monaghan 1994)
  - Together with ghost particles (Liu 2003)
  - Free slip



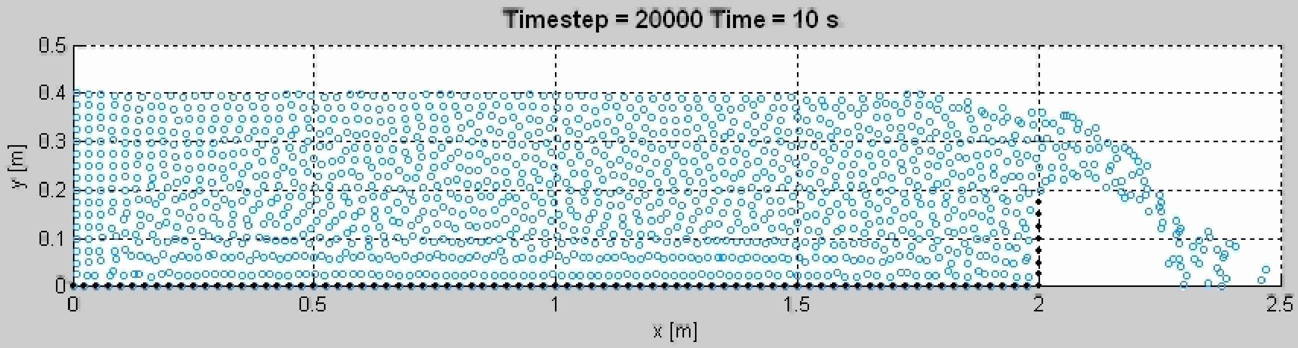
- Sharp weir schematic



- One measure of the water level is enough to calculate the discharge accurately (Rehbock formula)

- Left inflow, right clear overfall

- **Sharp weir in SPH (ghost particles are not shown)**
  - r_{ini} = 2.5 cm, 1300 particles on average
  - Inflow left / outflow right



  - Weir = boundary particles + ghost particles
  - Ghost particles outside the weir don’t interact with the water-jet

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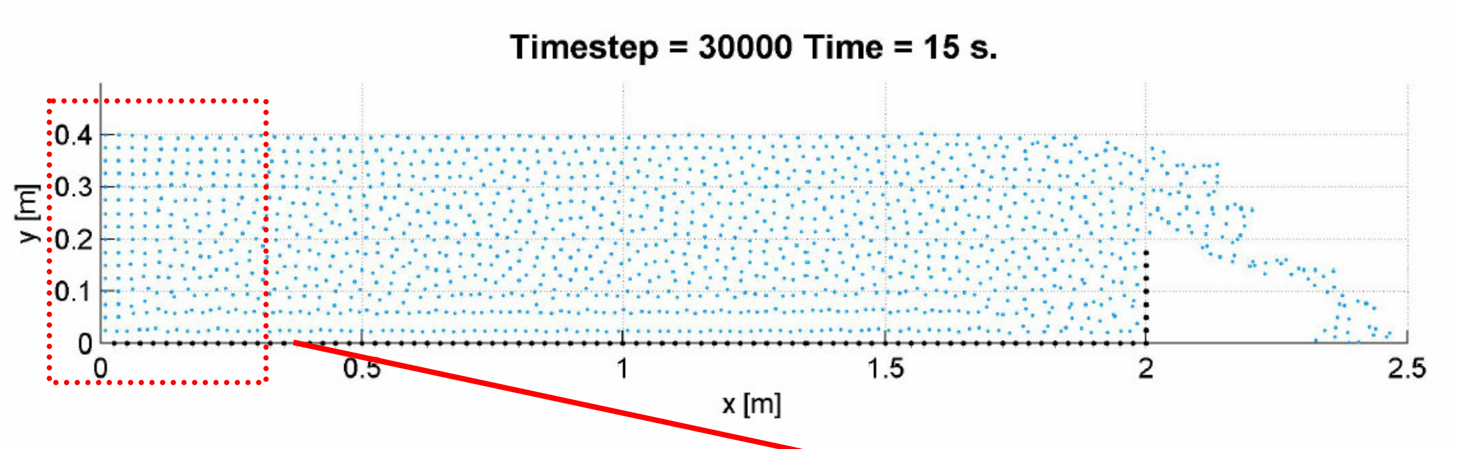
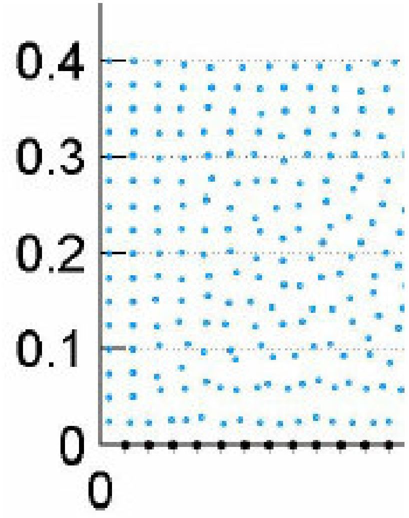
- **Inflow left**



  – Water level = 0.4 m
  – Vertical position of inflow particles is fixed
  – Horizontal inflow velocity is the same for all particles at inflow (not fixed!)
  – Ghost particles outside left (not shown)



- **Outflow right**
  – Water jet is simply cut off, particles below y=0 are removed

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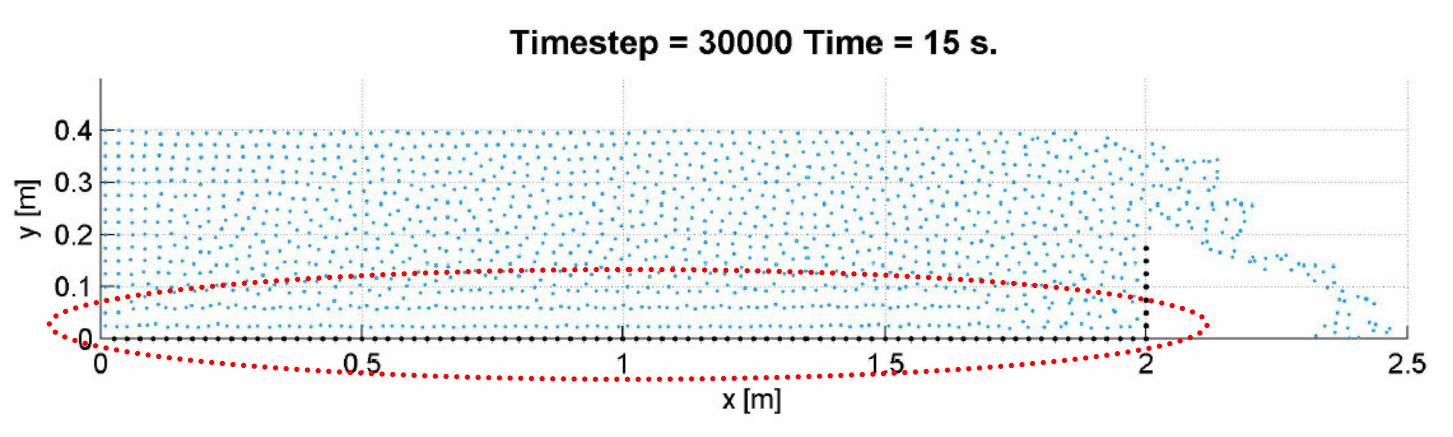
- **Boundary effect**



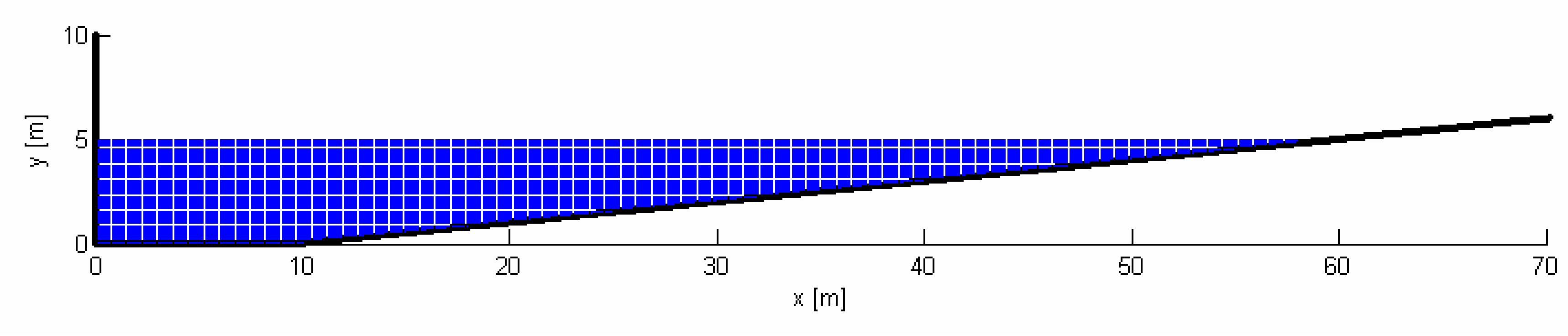
  - Even with ghost particles unphysical particle-lines occur
  - Possible reason: Density and pressure are mirrored at the bottom

- **Discharge**
  - $q_{Rehbock} = 0.2145\ m^2/s$
  - $q_{SPH} = 0.2260\ m^2/s$
  - Difference only 5 %

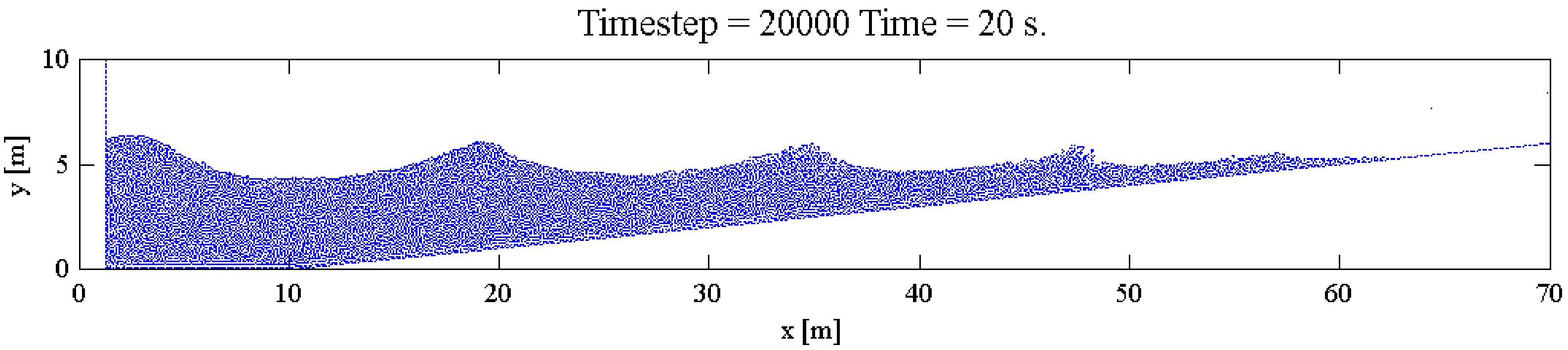
- **Waves on beach, like Monaghan (1994)**
- **Wavemaker left / slope 1:10 right**

- **Resolution = 2x resolution Monaghan**
- **r_{ini} = 0.1 m; 17000 particles**

- **Two simulations**
  1. Irribarren = 0.3 (spilling waves)
  2. Irribaren = 0.5 (plunging waves)

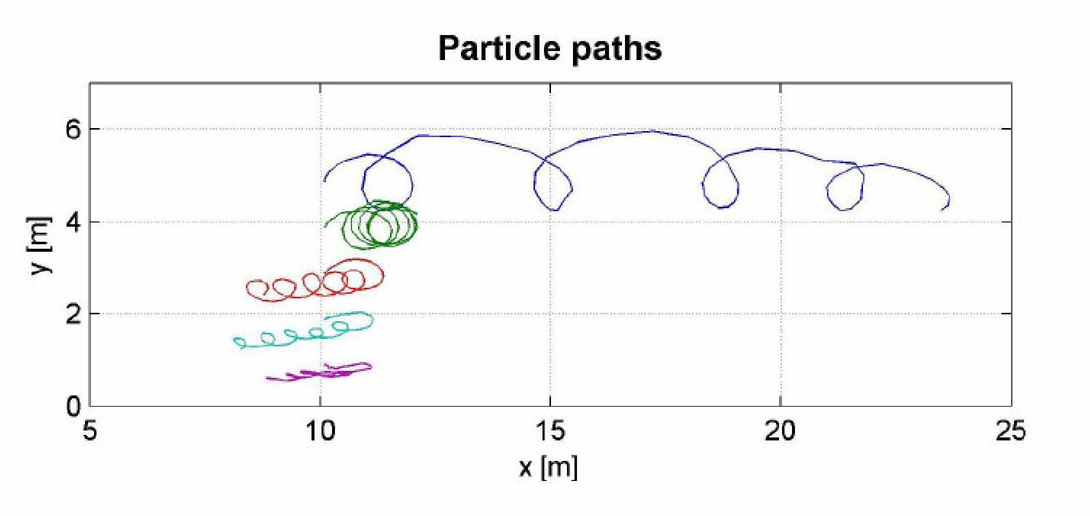


Spilling waves on a beach

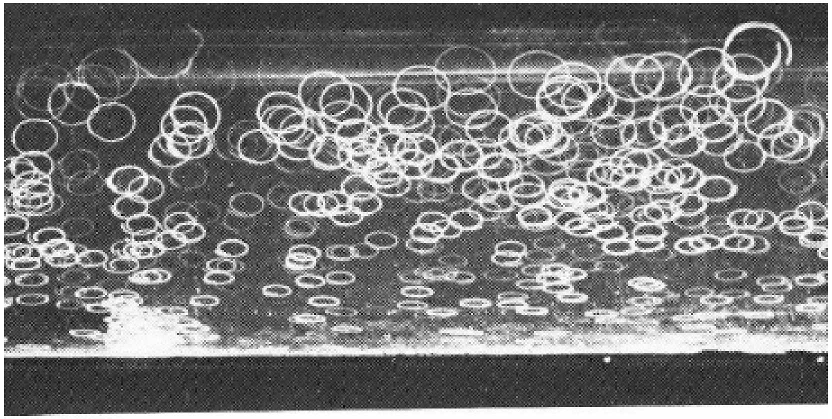


Spilling waves on a beach

- **Orbital motion**



  - Trajectories are clockwise ellipses
  - Circular near free surface, flattened near bottom
  - Drift right near surface
  - Drift left near bottom



No reflection: pure progressive waves

Plunging waves on a beach

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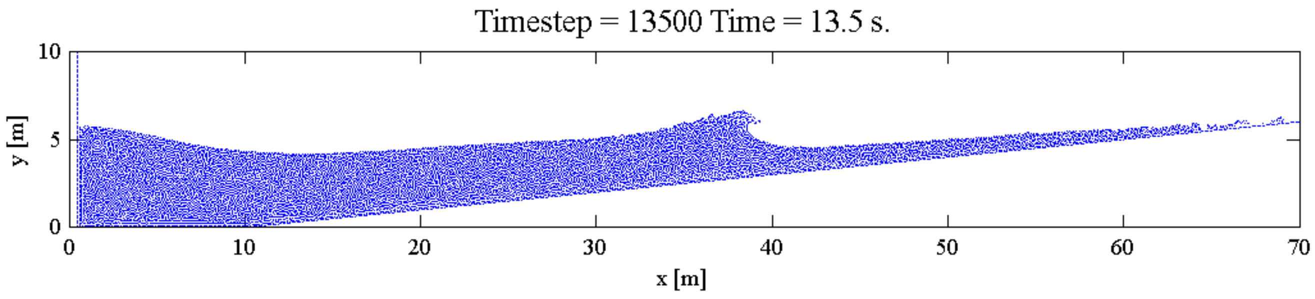
- **Moving animation on www.svasek.com**
  - See Hydraulic research → SPH



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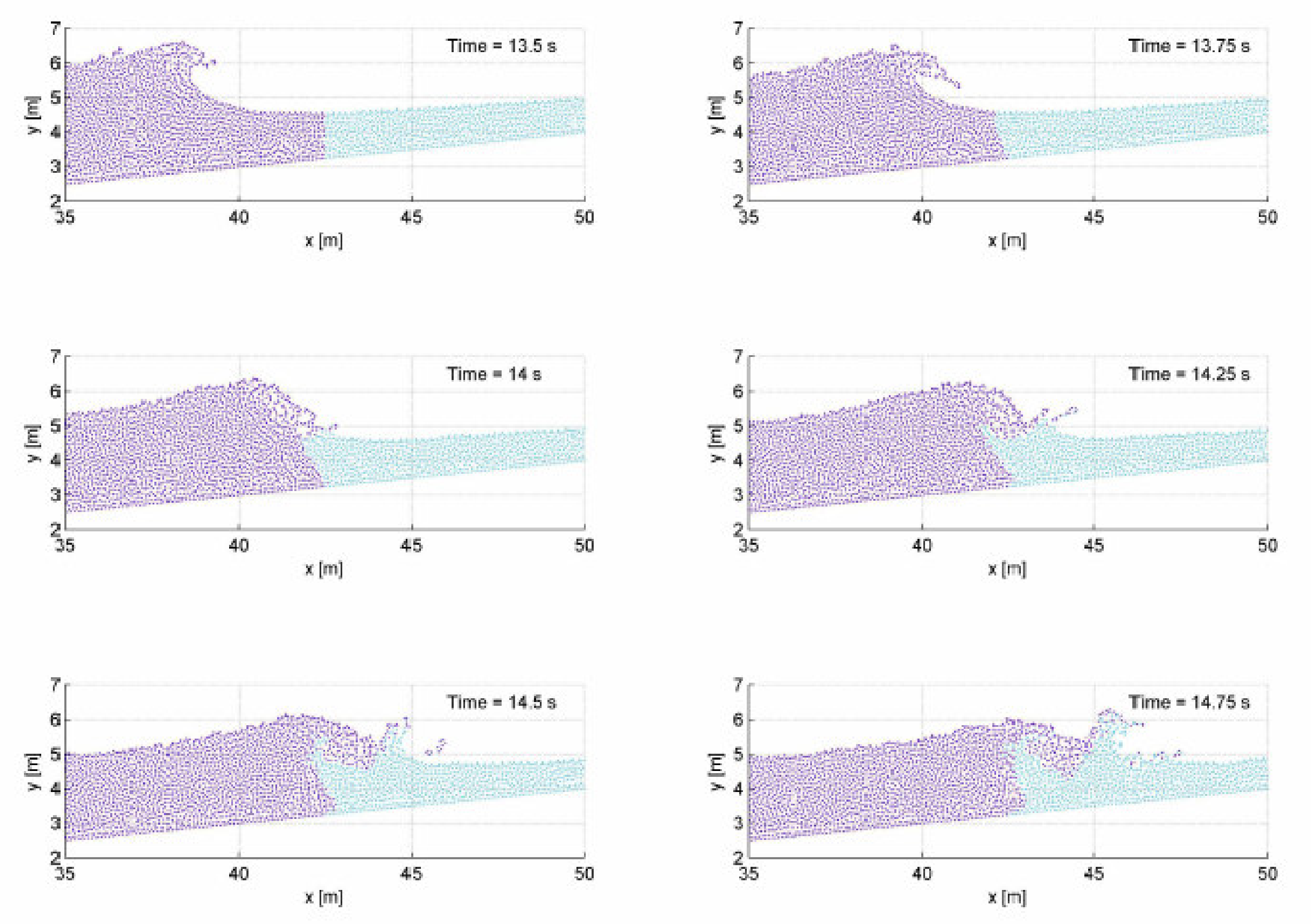
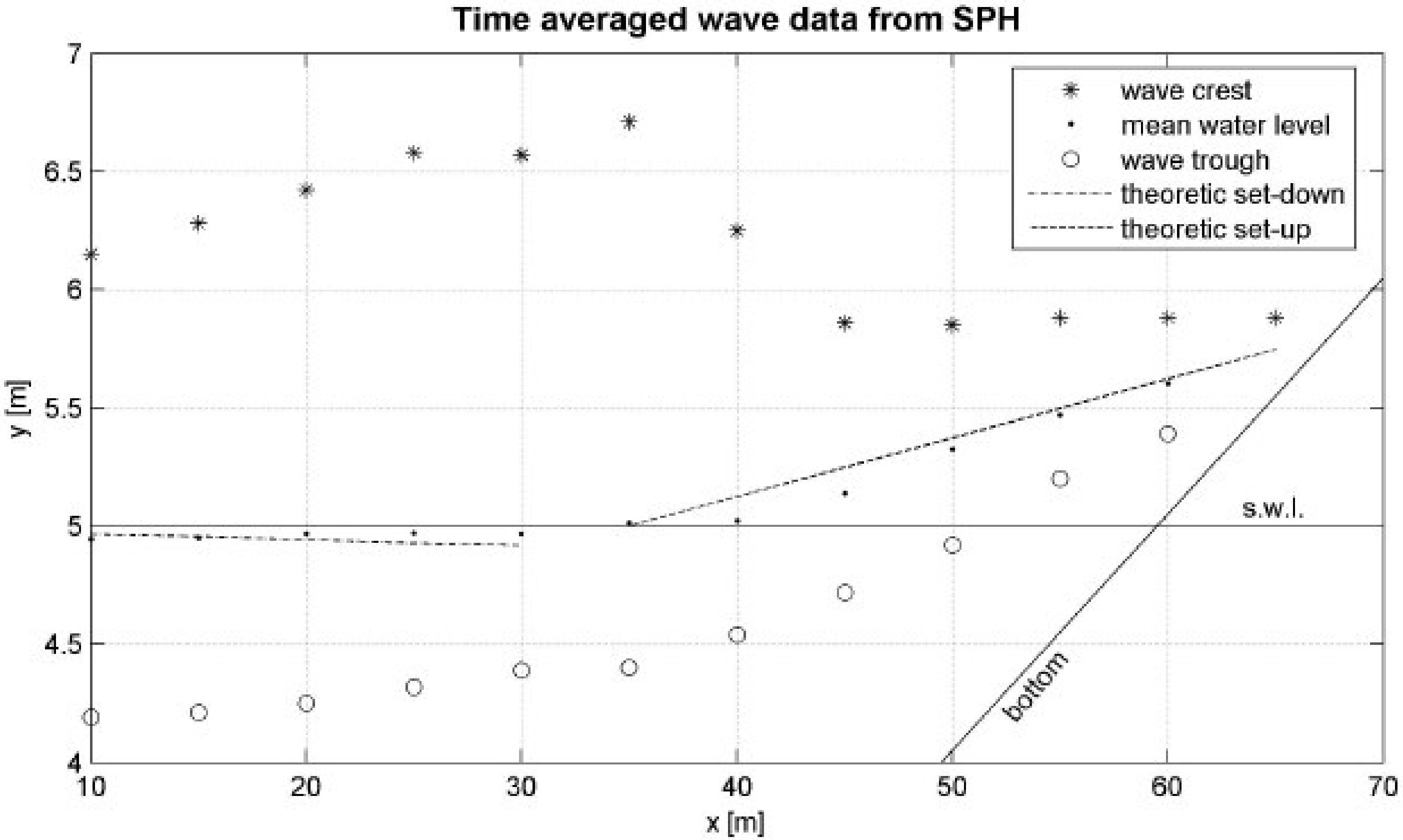
Close up plunging wave



- **Wave data from SPH**



– Theoretic set-down: $\bar{\eta} = -1/8 \frac{kH^2}{\sinh 2kd}$ Theoretic set-up: $\frac{d\bar{\eta}}{dx} = \frac{3/8\gamma^2}{1+3/8\gamma^2} \tan\alpha$

Conclusions

- Realistic flow configuration and discharge at sharp weir
- Simple inflow boundary with fixed water level and uniform inflow velocity is possible
- Wave behavior is correct

- Improvement on closed boundaries is welcome
- Positive wave speed error found
- Artificial viscosity gives a lot of dissipation