HYBRID RANS/LES : BLENDING MODELS AND BLENDING FILTERS

Massimo GERMANO

Dipartimento di Ingegneria Aeronautica e Spaziale
Politecnico di Torino - Italy

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School of MACE, The University of Manchester

OUTLINE

- **HYBRID RANS/LES MODELLING**
  - Practical motivations
  - Theoretical motivations
- **DIFFERENT APPROACHES**
  - Zonal decompositions
  - Universal modelling
  - Blending
- **BLENDING APPROACHES**
  - Blending models
  - Blending filters
- **CONCLUSIONS**

PRACTICAL MOTIVATIONS

- **REDUCTION OF THE COMPUTATIONAL COST**
  RANS : mean quantities at a moderate cost
  LES : higher order statistics at increased cost
- **HYBRID MODELLING : RANS/LES COUPLING**
  Tangential RANS/LES coupling
  Streamwise RANS/LES coupling

PRACTICAL MOTIVATIONS

- *The coupling of large eddy simulation with statistical turbulence models, i.e. Reynolds Averaged Navier Stokes models, is arguably the main strategy to drastically reduce computational cost for making LES affordable in a wide range of complex industrial applications*
  J. Fröhlich, D. von Terzi (2008) Hybrid LES/RANS methods for the simulation of turbulent flows, Progress in Aerospace Sciences, 44, 349-377

- *When attached boundary layers have a significant impact on the global flow dynamics, the use of hybrid RANS/lES methods remains the principal strategy to reduce computational cost compared to LES*
  Pierre Sagaut and Sebastien Deck, Large eddy simulation for aerodynamics: status and perspectives, Phil. Trans: R. Soc. A, 367, (2009), 2849-2860

THEORETICAL MOTIVATIONS

- **BASIC KNOWLEDGE OF TURBULENCE**
  RANS : statistical representation of the turbulence
  LES : filtered representation of turbulence
- **THE BASIC PROBLEM OF THE AVERAGE**
  Coupling different representations
  Coupling different models
  Coupling different codes
- **THE DIALOGUE BETWEEN RANS AND LES**

DIFFERENT APPROACHES

- **ZONAL INTERFACING**
- **UNIVERSAL MODELLING**
- **BLENDING**

  Blending Models

  Blending Filters

BLENDING MODELS

- The Prandtl RANS model
- The Smagorinsky LES model
- The two part model of Schumann (1975)
- The residual stress of Moin and Kim (1982)
- The two part model of Sullivan *et al* (1994)
- The blending viscosity models of Baggett (1998)
- The two velocities hybrid model of Uribe *et al* (2006)
- The weighted sum of LES and RANS models

BLENDING MODELS

The Prandtl RANS model

$$RANS \Longrightarrow \mathcal{R}(u_i) = \langle u_i \rangle_r$$

$$\tau_r(u_i, u_j) = \langle u_i u_j \rangle_r - \langle u_i \rangle_r \langle u_j \rangle_r$$

$$\tau_r(u_i, u_j) - \frac{\tau_r(u_k, u_k)}{3}\delta_{ij} = -2\nu_r \langle S_{ij} \rangle_r$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j}\right)$$

- $\nu_r = \Delta_r V_r$ is an eddy viscosity accounting for the turbulent dissipation
- Δ_r is a characteristic mixing length macro-scale
- V_r is a characteristic velocity macro-scale

BLENDING MODELS

The Smagorinsky LES model

$$LES \Longrightarrow \mathcal{L}(u_i) = \langle u_i \rangle_l$$

$$\tau_l(u_i, u_j) = \langle u_i u_j \rangle_l - \langle u_i \rangle_l \langle u_j \rangle_l$$

$$\tau_l(u_i, u_j) - \frac{\tau_l(u_k, u_k)}{3} \delta_{ij} = -2\nu_l \langle S_{ij} \rangle_l$$

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \right)$$

- $\nu_l = \Delta_l V_l$ is an eddy viscosity accounting for the local grid dissipation
- Δ_l is a characteristic length grid-scale
- V_l is a characteristic velocity grid-scale

BLENDING MODELS

The two part model of Schumann

$$HYBRID \Longrightarrow \mathcal{H}(u_i) = \langle u_i \rangle_h$$

$$\tau_h(u_i, u_j) = \langle u_i u_j \rangle_h - \langle u_i \rangle_h \langle u_j \rangle_h$$

$$\tau_h(u_i, u_j) - \frac{\tau_h(u_k, u_k)}{3}\delta_{ij} = -2\nu_h \left(\langle S_{ij} \rangle_h - \langle S_{ij} \rangle_r \right) - 2\nu_r \langle S_{ij} \rangle_r$$

- ν_h is an eddy viscosity accounting for locally isotropic turbulence related to the grid volume
- ν_r is a mixing length model

U. Schumann (1975) *Subgrid scale model for finite difference simulations of turbulent flows in plane channels and annuli*, Journal of Computational Physics, 18, 376-404

BLENDING MODELS

The residual-stress of Moin and Kim

$$\tau_h(u_i, u_j) - \frac{\tau_h(u_k, u_k)}{3}\delta_{ij} = -2\nu_h \left(\langle S_{ij}\rangle_h - \langle S_{ij}\rangle_r\right) - 2\nu_r \langle S_{ij}\rangle_r$$

- ν_h is an eddy viscosity accounting for locally isotropic turbulence related to the grid volume
- ν_r is a mixing length model, where the geometric scale is the filter width in the homogeneous spanwise direction

P. Moin and J. Kim (1982) *Numerical investigation of turbulent channel flow*, J. Fluid Mech., 118, 341-377

BLENDING MODELS

The two part model of Sullivan *et al*

$$\tau_h(u_i, u_j) - \frac{\tau_h(u_k, u_k)}{3}\delta_{ij} = -2\nu_h \gamma \langle S_{ij} \rangle_h - 2\nu_r \langle S_{ij} \rangle_r$$

- ν_h is a *fluctuating* eddy viscosity accounting for locally isotropic turbulence
- ν_r is a *mean field* eddy viscosity
- γ is an *isotropy factor* that controls the transition between SGS and ensemble average turbulence parameterizations

P. P. Sullivan, J. C. McWilliams and C. H. Moeng (1994) *A subgrid scale model for large eddy simulation of planetary boundary layer flows*, Boundary Layer Meteorology, 71, 247-276

BLENDING MODELS

The blending viscosity model of Baggett

$$\tau_h(u_i, u_j) - \frac{\tau_h(u_k, u_k)}{3}\delta_{ij} = -2[(1-k)\nu_h + k\nu_r]\langle S_{ij}\rangle_h$$

- ν_h is the dynamic Smagorinsky eddy viscosity
- ν_r is the eddy viscosity furnished by an external RANS simulation
- k is a blending function

J. S. Baggett (1998) *On the feasibility of merging LES with RANS for the near wall region of attached turbulent flows*, Center of Turbulence Research, Annual Research Briefs 1998, 267-276

BLENDING MODELS

The two velocities Hybrid RANS-LES Model of Uribe *et al*

$$\tau_h(u_i,u_j) - \frac{\tau_h(u_k,u_k)}{3}\delta_{ij} = -2\nu_s f_b\left(\langle S_{ij}\rangle_h - \langle S_{ij}\rangle_r\right) -2(1-f_b)\nu_r\langle S_{ij}\rangle_r$$

- ν_s is the standard Smagorinsky eddy viscosity
- ν_r is a RANS eddy viscosity related to the $\varphi - f$ RANS model
- f_b is a blending function depending on the ratio of the turbulent length scale to the filter width

J. C. Uribe (2006) *An industrial approach to near-wall turbulence modelling for unstructured finite volume methods*, PhD Thesis, University of Manchester, School of MACE, Chapter 10

BLENDING MODELS

A weighted sum of LES and RANS models

$$\tau_h(u_i, u_j) = k\tau_r(u_i, u_j) + (1-k)\tau_l(u_i, u_j)$$

What does it mean exactly ?

BLENDING MODELS

$$
\begin{aligned}
RANS &\Longrightarrow \mathcal{R}(u_i) = \langle u_i \rangle_r \\
LES &\Longrightarrow \mathcal{L}(u_i) = \langle u_i \rangle_l \\
HYBRID &\Longrightarrow \mathcal{H}(u_i) = \langle u_i \rangle_h
\end{aligned}
$$

$$
\begin{aligned}
\tau_r(u_i, u_j) &= \langle u_i u_j \rangle_r - \langle u_i \rangle_r \langle u_j \rangle_r \sim M_r(\langle u_i \rangle_r, \langle u_i \rangle_r) \\
\tau_l(u_i, u_j) &= \langle u_i u_j \rangle_l - \langle u_i \rangle_l \langle u_j \rangle_l \sim M_l(\langle u_i \rangle_l, \langle u_i \rangle_l) \\
\tau_h(u_i, u_j) &= \langle u_i u_j \rangle_h - \langle u_i \rangle_h \langle u_j \rangle_h \sim M_h(\langle u_i \rangle_h, \langle u_i \rangle_h)
\end{aligned}
$$

BLENDING MODELS

$$\tau_h(u_i, u_j) = k\tau_r(u_i, u_j) + (1-k)\tau_l(u_i, u_j)$$

$$M_h(\langle u_i\rangle_h, \langle u_i\rangle_h) \sim kM_r(\langle u_i\rangle_r, \langle u_i\rangle_r) + (1-k)M_l(\langle u_i\rangle_l, \langle u_i\rangle_l)$$

$$\begin{aligned} \langle u_i\rangle_h &\Longrightarrow \langle u_i\rangle_r \\ \langle u_i\rangle_h &\Longrightarrow \langle u_i\rangle_l \end{aligned}$$

BLENDING MODELS

$$\tau_h(u_i, u_j) = k\tau_r(u_i, u_j) + (1-k)\tau_l(u_i, u_j)$$

$$M_h(\langle u_i\rangle_h, \langle u_i\rangle_h) \sim kM_r(\langle u_i\rangle_r, \langle u_i\rangle_r) + (1-k)M_l(\langle u_i\rangle_h, \langle u_i\rangle_h)$$

$$\langle u_i\rangle_h \Longrightarrow \langle u_i\rangle_r$$

BLENDING MODELS

$$\tau_h(u_i, u_j) = k\tau_r(u_i, u_j) + (1-k)\tau_l(u_i, u_j)$$

$$M_h(\langle u_i\rangle_h, \langle u_i\rangle_h) \sim kM_r(\langle u_i\rangle_h, \langle u_i\rangle_h) + (1-k)M_l(\langle u_i\rangle_h, \langle u_i\rangle_h)$$

BLENDING FILTERS

- **THE FILTERING APPROACH**
  Explicit and operational filtering
  The subgrid central moments
- **THE SUBGRID STRESS ASSOCIATED TO THE PRODUCT OF TWO FILTERS**
  The dynamic model
- **THE SUBGRID STRESS ASSOCIATED TO THE SUM OF TWO FILTERS**
  The hybrid RANS/LES additive model

BLENDING FILTERS

The filtering approach

- *In practice it is most often used for conceptual reasons than as a precise algorithmic construction.*

  J. Fröhlich, W. Rodi Introduction to large-eddy simulation of turbulent flows, in: Launder B, Sandham N, editors, Closure strategies for turbulent and transitional flows, Cambridge University Press, (2002), pag. 274

- *The filtered equations are isomorphic to the original equations, (the filtering paradox).*

  J. L. Guermond, J. T. Oden and S. Prudhomme, Mathematical Perspectives on Large Eddy Simulation Models for Turbulent Flows, J. Math. Fluid Mech. 6 (2004), 194-248

THE EXPLICIT FILTERING FORMULATION

$$\frac{\partial \langle u_n \rangle_f}{\partial x_n} = 0 \quad ; \quad \frac{\partial \langle u_i \rangle_f}{\partial t} + \frac{\partial \langle u_i u_n \rangle_f}{\partial x_n} = -\frac{\partial \langle p \rangle_f}{\partial x_i} + \nu \frac{\partial^2 \langle u_i \rangle_f}{\partial x_n \partial x_n}$$

$$\langle u_i \rangle_f = \mathcal{F}[u_i] = \int_\varepsilon u_i(y_k, t) F(x_k - y_k) dy_1 dy_2 dy_3$$

$$\langle p \rangle_f = \mathcal{F}[p] = \int_\varepsilon p(y_k, t) F(x_k - y_k) dy_1 dy_2 dy_3$$

$$\langle u_i u_n \rangle_f = \mathcal{F}[u_i u_n] = \int_\varepsilon u_i(y_k, t) u_n(y_k, t) F(x_k - y_k) dy_1 dy_2 dy_3$$

THE OPERATIONAL FILTERING FORMULATION

$$\mathcal{C}\left[\mathbf{u}\right] = \frac{\partial u_n}{\partial x_n} = 0$$

$$\mathcal{C}\left[\mathcal{F}[\mathbf{u}]\right] = q \quad ; \quad q = \mathcal{C}\left[\mathcal{F}[\mathbf{u}]\right] - \mathcal{F}[\mathcal{C}\left[\mathbf{u}\right]]$$

$$\mathcal{N}_i\left[\mathbf{u}, p\right] = \frac{\partial u_i}{\partial t} + \frac{\partial u_i u_n}{\partial x_n} + \frac{\partial p}{\partial x_i} - \nu \frac{\partial^2 u_i}{\partial x_n \partial x_n} = 0$$

$$\mathcal{N}_i\left[\mathcal{F}[\mathbf{u}], \mathcal{F}[p]\right] = f_i \quad ; \quad f_i = \mathcal{N}_i\left[\mathcal{F}[\mathbf{u}], \mathcal{F}[p]\right] - \mathcal{F}\left[\mathcal{N}_i[\mathbf{u}, p]\right]$$

THE STATISTICAL CENTRAL MOMENTS

$$
\begin{aligned}
R_{AB} &= \langle AB\rangle_r - \langle A\rangle_r\langle B\rangle_r \\
R_{ABC} &= \langle ABC\rangle_r - \langle A\rangle_r R_{BC} - \langle B\rangle_r R_{CA} - \langle C\rangle_r R_{AB} \\
&- \langle A\rangle_r\langle B\rangle_r\langle C\rangle_r \\
R_{ABCD} &= \langle ABCD\rangle_r - \langle A\rangle_r R_{BCD} \\
&- \langle B\rangle_r R_{CDA} - \langle C\rangle_r R_{DAB} - \langle D\rangle_r R_{ABC} \\
&- \langle A\rangle_r\langle B\rangle_r R_{CD} - \langle A\rangle_r\langle C\rangle_r R_{BD} \\
&- \langle A\rangle_r\langle D\rangle_r R_{BC} - \langle B\rangle_r\langle C\rangle_r R_{AD} \\
&- \langle B\rangle_r\langle D\rangle_r R_{AC} - \langle C\rangle_r\langle D\rangle_r R_{AB} \\
&- \langle A\rangle_r\langle B\rangle_r\langle C\rangle_r\langle D\rangle_r
\end{aligned}
$$

THE STATISTICAL CENTRAL MOMENTS

$$\langle\langle A\rangle_r\rangle_r = \langle A\rangle_r \quad , \quad \langle A\langle B\rangle_r\rangle_r = \langle A\rangle_r\langle B\rangle_r$$

$$A = \langle A\rangle_r + A' \quad , \quad B = \langle B\rangle_r + B' \quad , \quad \cdots$$

$$\begin{aligned} R_{AB} &= \langle A'B'\rangle_r \\ R_{ABC} &= \langle A'B'C'\rangle_r \\ R_{ABCD} &= \langle A'B'C'D'\rangle_r \end{aligned}$$

THE SUBGRID CENTRAL MOMENTS

$$
\begin{aligned}
\tau_{AB} &= \langle AB\rangle_f - \langle A\rangle_f\langle B\rangle_f \\
\tau_{ABC} &= \langle ABC\rangle_f - \langle A\rangle_f\tau_{BC} - \langle B\rangle_f\tau_{CA} \\
&- \langle C\rangle_f\tau_{AB} - \langle A\rangle_f\langle B\rangle_f\langle C\rangle_f \\
\tau_{ABCD} &= \langle ABCD\rangle_f - \langle A\rangle_f\tau_{BCD} - \langle B\rangle_f\tau_{CDA} \\
&- \langle C\rangle_f\tau_{DAB} - \langle D\rangle_f\tau_{ABC} - \langle A\rangle_f\langle B\rangle_f\tau_{CD} \\
&- \langle A\rangle_f\langle C\rangle_f\tau_{BD} - \langle A\rangle_f\langle D\rangle_f\tau_{BC} - \langle B\rangle_f\langle C\rangle_f\tau_{AD} \\
&- \langle B\rangle_f\langle D\rangle_f\tau_{AC} - \langle C\rangle_f\langle D\rangle_f\tau_{AB} - \langle A\rangle_f\langle B\rangle_f\langle C\rangle_f\langle D\rangle_f
\end{aligned}
$$

THE SUBGRID STRESS ASSOCIATED TO THE PRODUCT OF TWO FILTERS

$$\mathcal{P} = \mathcal{G}\mathcal{F} \quad ; \quad \langle A \rangle_p = \langle \langle A \rangle_f \rangle_g$$

$$\begin{aligned} \tau_p(A,B) &= \langle AB \rangle_p - \langle A \rangle_p \langle B \rangle_p = \\ &= \langle \tau_f(A,B) \rangle_g + \tau_g(\langle A \rangle_f, \langle B \rangle_f) \end{aligned}$$

$$\begin{aligned} \tau_f(A,B) &= \langle AB \rangle_f - \langle A \rangle_f \langle B \rangle_f \\ \tau_g(\langle A \rangle_f, \langle B \rangle_f) &= \langle \langle A \rangle_f \langle B \rangle_f \rangle_g - \langle \langle A \rangle_f \rangle_g \langle \langle B \rangle_f \rangle_g \end{aligned}$$

Germano M. *Averaging Invariance of the Turbulent Equations and Similar Subgrid Modeling*, CTR Manuscript 116, 1990

THE DYNAMIC MODEL

- $\mathcal{F}$ is the LES filtering operator
- $\mathcal{G}$ is an explicit test filtering operators

The dynamic model : The subgrid stress model associated to the product $\mathcal{P}$ of a test filtering operator $\mathcal{G}$ with the LES filtering operator $\mathcal{F}$ must be consistent with the subgrid stress model associated to $\mathcal{F}$.

Germano M., Piomelli U., Moin P. and Cabot W.H. 1991 *A dynamic subgrid-scale eddy viscosity model* Phys.Fluids A 3, 1760-1765

THE SUBGRID STRESS ASSOCIATED TO THE SUM OF TWO FILTERS

$$\mathcal{S} = k\mathcal{F} + (1-k)\mathcal{G} \quad ; \quad \langle A \rangle_s = k\langle A \rangle_f + (1-k)\langle A \rangle_g$$

$$\begin{aligned} \tau_f(A,B) &= \langle AB \rangle_f - \langle A \rangle_f \langle B \rangle_f \\ \tau_g(A,B) &= \langle AB \rangle_g - \langle A \rangle_g \langle B \rangle_g \end{aligned}$$

$$\begin{aligned} \tau_s(A,B) &= k\tau_f(A,B) + (1-k)\tau_g(A,B) + \\ &+ k(1-k)(\langle A \rangle_f - \langle A \rangle_g)(\langle B \rangle_f - \langle B \rangle_g) \end{aligned}$$

Germano M. 2004 *Properties of the hybrid RANS/LES filter*, Theoret. Comput. Fluid Dynamics, 17, pag. 225-231
Germano M., Sagaut P. 2006 *Formal properties of the additive RANS/DNS filter*, Proc. of the Sixth Int. ERCOFTAC Workshop on DNS and LES, Springer, pag. 127-134

THE HYBRID RANS/LES ADDITIVE MODEL

The associated additive stress

$$\mathcal{H} = k\mathcal{L} + (1-k)\mathcal{R}$$

$$\langle u_i \rangle_h = k\langle u_i \rangle_l + (1-k)\langle u_i \rangle_r$$

$$\begin{aligned}\tau_h(u_i, u_j) &= k\tau_l(u_i, u_j) + (1-k)\tau_r(u_i, u_j) + \\ &+ k(1-k)(\langle u_i \rangle_l - \langle u_i \rangle_r)(\langle u_j \rangle_l - \langle u_j \rangle_r)\end{aligned}$$

$$\begin{aligned}\tau_l(u_i, u_j) &= \langle u_i u_j \rangle_l - \langle u_i \rangle_l \langle u_j \rangle_l \\ \tau_r(u_i, u_j) &= \langle u_i u_j \rangle_r - \langle u_i \rangle_r \langle u_j \rangle_r\end{aligned}$$

THE HYBRID RANS/LES ADDITIVE MODEL

The commutation with the derivatives

$$\mathcal{H} = k\mathcal{L} + (1-k)\mathcal{R} \quad ; \quad k = k(x_i)$$

$$\mathcal{H}\mathcal{D}_i = \mathcal{D}_i\mathcal{H} - \frac{\partial k}{\partial x_i}(\mathcal{L} - \mathcal{R}) \quad ; \quad \mathcal{D}_i = \frac{\partial}{\partial x_i}$$

$$\frac{\partial u_j}{\partial x_j} = 0 \quad ; \quad \left\langle \frac{\partial u_j}{\partial x_j} \right\rangle_h = 0$$

$$\frac{\partial \langle u_j \rangle_h}{\partial x_j} = \frac{\partial k}{\partial x_j}(\langle u_j \rangle_l - \langle u_j \rangle_r)$$

THE HYBRID RANS/LES ADDITIVE MODEL

The reconstruction of $(\langle u_i \rangle_l - \langle u_i \rangle_r)$

$$\mathcal{H} = k\mathcal{L} + (1-k)\mathcal{R}$$

If $\quad \mathcal{R}\mathcal{H} = \mathcal{R} \quad$ then

$$\langle u_i \rangle_r = \langle \langle u_i \rangle_h \rangle_r$$

$$\langle u_i \rangle_l = \frac{\langle u_i \rangle_h - (1-k)\langle u_i \rangle_r}{k}$$

$$\langle u_i \rangle_l - \langle u_i \rangle_r = \frac{\langle u_i \rangle_h - \langle \langle u_i \rangle_h \rangle_r}{k}$$

THE HYBRID RANS/LES ADDITIVE MODEL

First results

- Martín Sánchez-Rocha, Suresh Menon, The compressible hybrid RANS/LES formulation using an additive operator, Journal of Computational Physics 228, (2009), 2037-2062

  .....these additional terms play a fundamental role in compensating for the turbulence that is neither modeled nor resolved in the transition region between RANS and LES

- Bernie Rajamani and John Kim, A Hybrid-Filter Approach to Turbulence Simulation, Flow Turbulence Combust., 85, (2010), 421-441

  Despite some issues associated with the numerical implementation, good results were obtained for the mean velocity and skin friction coefficient. The mean velocity profile did not have an overshoot in the logarithmic region for most blending functions, confirming that proper energy transfer from the RANS to the LES region was a key to successful hybrid models

CONCLUSIONS

- The operational filtering approach remains a useful formalization of LES.
- The study of the formal properties of the operational filtering approach can produce useful suggestions to the analysis and the validation of LES data and to the study and the improvement of subgrid and hybrid models.
- The improvement of the dialogue between RANS and LES is fundamental to the progress of the turbulence research.
- The development of a subgrid model and the validation of a LES code requires a lot of work and cooperation between different people and different experimental, numerical and theoretical techniques.