FLOW AND HEAT-TRANSFER MODELLING OF THREE-DIMENSIONAL JET IMPINGEMENT ON A CONCAVE SURFACE

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Introduction

- **Motivation**
  Internal cooling of the leading edge of gas-turbine blades.

- **Challenge**
  The flow displays many challenging features.
  - Impingement
  - Surface Curvature
  - Jet Interaction
  - Orthogonal Rotation

- **Focus**
  Jet impingement onto a concave surface.

- **Objective**
  To examine the performance of non-linear eddy-viscosity models and recently-developed wall-function approach.
Case Studied

- Flow and heat-transfer measurements from a row of five circular jets impinging onto a heated, semi-circular, passage wall.

**Experimental Study, Iacovides et al (2005).**

- $s/d_J=4$, $R/d_J=3.125$
- $Re \equiv V_J d_J / \nu$
  - $9,400 \& 15,000$
- $Ro \equiv \Omega d_J / V_J$
  - $-0.18 \text{ to } +0.18$
- $Pr = 6$

**Flow Visualisation Images**
Turbulence Modelling

• Full second-moment closures, and a complete resolution of the near-wall viscosity-affected layer, offer potentially the most reliable modelling route.

• However, in large, complex, 3-D flows the computational cost of such an approach can be prohibitive.

• Computations have thus been performed using high-Reynolds-number turbulence models, with wall-functions to bridge the viscous sub-layer.

• Linear and non-linear eddy-viscosity models have been used.

• The linear k-ε model is known to give poor predictions of many relevant flow features – including impingement, flow curvature, . . .

• Most calculations here have employed a non-linear eddy-viscosity model, developed in Manchester (UMIST) by Suga (1995).
Turbulence Modelling, Non-Linear EVM

- Two important features of the non-linear EVM tested are:
  - Turbulent viscosity $\nu_t = c_\mu k^2/\varepsilon$ with $c_\mu$ a function of local strain rates.
  - A cubic non-linear stress-strain relation

\[
\overline{u_i u_j} = (2/3)k \delta_{ij} - \nu_t S_{ij} + c_1 \frac{\nu_t k}{\varepsilon} (S_{ik} S_{kj} - (1/3) S_{kl} S_{kl} \delta_{ij}) \\
+ c_2 \frac{\nu_t k}{\varepsilon} (\Omega_{ik} S_{kj} + \Omega_{jk} S_{ki}) + c_3 \frac{\nu_t k}{\varepsilon} (\Omega_{ik} \Omega_{jk} - (1/3) \Omega_{lk} \Omega_{lk} \delta_{ij}) \\
+ c_4 \frac{\nu_t k^2}{\varepsilon^2} (S_{ki} \Omega_{lj} + S_{kj} \Omega_{li}) S_{kl} + c_6 \frac{\nu_t k^2}{\varepsilon^2} S_{ij} S_{kl} S_{kl} + c_7 \frac{\nu_t k^2}{\varepsilon^2} S_{ij} \Omega_{kl} \Omega_{kl}
\]

- Where

\[
S_{ij} = \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \\
\Omega_{ij} = \left( \frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right) - 2 \varepsilon_{ijk} \Omega_k
\]
Turbulence Modelling, Near-Wall Modelling – 1

- Standard wall functions assume the near-wall velocity follows the logarithmic profile
  \[ U^* = \frac{1}{\kappa^*} \log(E^*y^*) \text{ with } U^* = U_k p^{1/2} \left( \frac{\tau_w}{\rho} \right) \text{ and } y^* = y_k p^{1/2} / \nu \]

- In complex flows, such as the present, the real flow will not exhibit such a behaviour.

- A more advanced scheme has been developed in Manchester (Craft et al, 2002), denoted here as the AWF (Analytic Wall Function).

- This assumes a distribution for \( \mu_t \) across the near-wall cell:
  \[ \mu_t = 0 \quad \text{for } 0 < y < y_v \]
  \[ \mu_t = \mu_c c_\mu (y^* - y_v^*) \quad \text{for } y_v < y < y_n \]
The simplified temperature equation across the near-wall cell becomes:

\[
\begin{align*}
\frac{\partial}{\partial y} \left( \frac{\mu}{Pr} \frac{\partial T}{\partial y} \right) &= C_{th1} \quad \text{for } 0 < y < y_v \\
\frac{\partial}{\partial y} \left( \mu \left[ \frac{1}{Pr} + c_l e (y^* - y_v^*) / Pr_l \right] \frac{\partial T}{\partial y} \right) &= C_{th2} \quad \text{for } y_v < y < y_n
\end{align*}
\]

Approximations for convective terms $C_{th1}$ and $C_{th2}$ allow one to analytically integrate the above equations and obtain the wall temperature, or heat flux.

A similar approach can be employed for the momentum equations.

AWF development considered wall-parallel flows, approximating only the wall-parallel convection $C_{th}$.
Turbulence Modelling, Near-Wall Modelling – 3
Treatment of Convection

- Here, due to the increased importance of new-wall convection, a more rigorous approach is developed for both T and U.
- Both the wall normal and wall parallel convection are separately evaluated over each layer, through numerical integration.
- During these integrations the wall parallel velocity U and the wall normal gradient, $\partial T/\partial y$ obtained from the analytical solutions.

\[
\left( U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} \right)_1 = \frac{1}{y_v} \int_{0}^{y_v} \left( U_1 \left( \frac{T_e - T_w}{\Delta x} \right) + V \frac{\partial T_1}{\partial y} \right) dy
\]

\[
\left( U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} \right)_2 = \frac{1}{y_n - y_v} \int_{y_v}^{y_n} \left( U_2 \left( \frac{T_e - T_w}{\Delta x} \right) + V \frac{\partial T_2}{\partial y} \right) dy
\]

- Assumed variation for wall normal velocity, V.

- When wall normal velocity away from the wall: $C_{Tn1} = C_{Tn2} = 0$
Craft et al (2002) suggested an empirical correction method that adjusts the cell-averaged turbulence energy dissipation rate in the near-wall cell by employing a correction factor, $F_\varepsilon$, as follows:

$$\bar{\varepsilon}_{\text{new}} = F_\varepsilon \bar{\varepsilon}_{\text{original}}$$

$$F_\varepsilon = \begin{cases} 
1.0 + 1.5 \left\{ 1.0 - \exp\left[-6.9 \left( \lambda - 0.98 \right) \right]\right\} \\
\times \left\{ 1.0 - \exp\left[-193.0 \left( \max(\alpha,0) \right)^2 \right]\right\}, & \lambda \geq 1.0 \\
1.0 - \left(1.0 - F_{\varepsilon 0}\right) \left[ 1.0 - \exp\left(-\frac{1-\lambda}{\lambda}\right) \right] \\
\times \left\{ 1.0 - \exp\left[-11.1 \left( \max(\gamma,0) \right)^2 \right]\right\}, & \lambda < 1.0 
\end{cases}$$

- For Numerical Stability, the parameter $\lambda$ in under-relaxed.
Numerical Treatment

• An in-house finite volume solver (STREAM) has been employed.
• This uses a multi-block structured grid arrangement.
• The SIMPLE pressure-correction method is used, with Rhie & Chow interpolation for mass fluxes.
• The UMIST convection scheme is applied for mean quantities and first order upwind for turbulence quantities.
• Thin boundary layer and uniform turbulent levels at jet inlets, match measurements close to the jet inlets.
• Initial calculations for only a single jet, with symmetry conditions at the mid-planes between jets.
• Subsequent computations for the entire 5-jet domain.
Grid Dependence Tests

- Grids containing 1/4 million, 1 million and 1.2 million cells have been tested.

- Nusselt number profiles at $z/D=0$ show no sensitivity to grid size.
Results – Mean Velocity Vectors

• Dynamic field results are fairly insensitive to the wall-function adopted.

• Wall jet collision at the symmetry plane results in a downwash.

• Calculations broadly reproduce the measured flow features.
• Both models underestimate the development of the incoming jet.
• Non-linear model returns higher peak values across the shear layers.
• Turbulence levels close to the jet inlet are reasonably well predicted, but are under predicted in the mid-passage region.
• Predicted Nu levels in reasonable agreement with those measured.

• The introduction of the AWF and of the non-linear k-ε results in improved predictions of the shape of the peak Nu regions above each jet and also to the prediction of the secondary peaks between stagnation points.
- Introduction of the AWF leads to the prediction of:
  - The correct peak Nu levels at the stagnation points
  - The secondary peaks in Nu.
- Nu levels between stagnation points underestimated
Effects of Rotation

Local Nusselt Number

Stationary Case

Clockwise Rotation, Ro = -0.18

Jet Development
Effects of Rotation

Secondary motion above Jet 3

Jet 3 development within symmetry plane

Rotation generates secondary motion, normal to the jet axis, which is reproduced by the predictions.

Predictions show increase in jet spreading rate with rotation.

Turbulence Intensity over Jet 3

Increase in turbulence with rotation not reproduced by computations.
Concluding Remarks

For the Stationary Case

- The complex dynamic field is reasonably well predicted by the non-linear model, although turbulence levels are under-predicted in the mid-passage region.

- Standard wall-functions give broadly correct average heat transfer levels, but fail to capture the distribution shape.

- The AWF, with a more refined 2-layer treatment of convective terms, improves the Nusselt number predictions, but still under-predicts heat transfer in the downwash regions.

For the rotating Case

- The development of Coriolis-induced secondary motion, normal to the jets is well reproduced.
The secondary motion
(a) Twists of the axes of the elliptical regions of peak Nusselt number, which is consistent with what happens to some of the peak Nu regions in the experiment.
(b) Increases the spreading rate of the jets, but not to the same degree as that suggested by the measurements.

The under-prediction of the jets’ spreading rates appears to be responsible for the failure to predict the disappearance of some impingement regions and associated the high Nusselt number levels.

The lower turbulence levels predicted at the jet inlets is a probable cause.

Current efforts focused on extending the solution domain to the chamber upstream of the injection slots.

Time-dependent flow computations may also be necessary.