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HEAT TRANSFER MODEL BASED ON THE IDEAS AND EQUATIONS OF STATISTICAL ENERGY ANALYSIS APPLIED TO RADIATION HEAT FLOW IN ELECTRIC FURNACE FOR MELTING SCRAP ALUMINIUM AND BILLET

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ABSTRACT

This paper describes the modeling of the superheating capability of an electric furnace system. The furnace design is radical in that it uses electric radiant heating elements to make use of cheap nuclear and geothermal power. The aim of the work was to create a mathematical model to evaluate conditions under which the superheat performance is likely to be met or exceeded. The model predicted that the furnace has enough heating power, subject to the correct preheating being applied first.

1 INTRODUCTION

Optimal design of furnace equipment is an important issue, given the cost of energy, cost of components and the need for economic operating procedures. Dynamic modeling offers the chance to evaluate factors affecting dynamic performance without costly testing. The benefits of the models outweigh the effort needed to build and test them.

In super heat mode, an electrically heated tilting furnace, for use with aluminium, must attain a melt temperature rise of 30 degrees C per hour. The attainment of this was both cost sensitive and contractually required.

To provide the business with some comfort that design and installation would go smoothly, mathematical calculations based on traditional methods had been completed.

To complement and check these methods, and using tools and techniques used in automotive development, the author created a dynamic computer simulation. This simulation was used to evaluate the superheat performance under a range of operating and design conditions.

2 THEORY

After consideration of the ideas and equations of statistical energy analysis [1], a simplified furnace model (Figure 5) was chosen consisting of two thermal bodies with heat capacity:

- The hearth and melt
- The roof, walls and heating elements

This seemed reasonable for a model of super heating, where the roof has been preheated by the elements, and the hearth contains molten metal.

Heat exchange between the thermal bodies is by radiation exchange, governed by the Stefan-Boltzmann law. Heat is lost separately from both thermal bodies to ambient, by conduction through the furnace walls. The thermal conductivity of both these leakage paths was provided by separate static finite difference models of the furnace walls.

The model chosen was therefore a hybrid: it used static calculations for the conduction losses, and dynamic calculations for the radiation exchanges, giving a dynamic model of temperature changes.

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The emissivity of the bodies required careful consideration [2]. The model assumed local thermal equilibrium with emissivity () and absorptivity () equal. There are two problems with this:

- The radiating body and the receiving body are not at the same temperature.
- The radiating body and the receiving body have different surface properties.

Given these assumptions, it was felt that the model could have value.

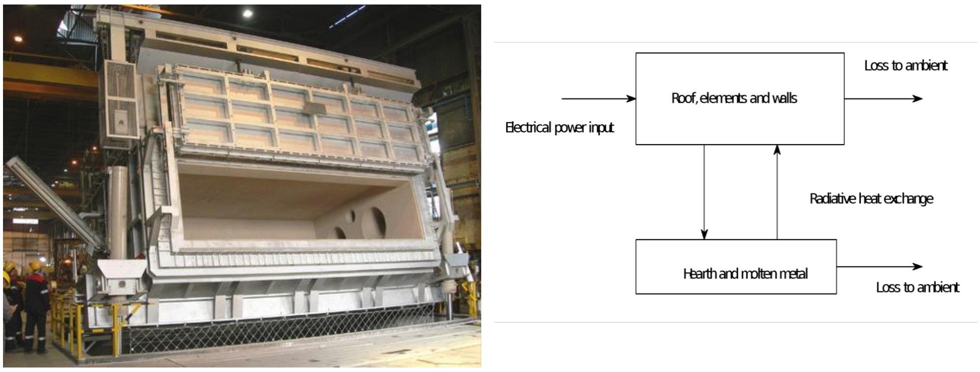

Figure 1: a) Furnace in tilt for pouring of the melt and b) 2 body energy flow model.

3 MATHEMATICAL FORMULAE

The two body problem was formulated for numerical solution in MathCAD [3]. The model was tested with a power off condition.

$$m_0 \cdot s_0 \cdot \frac{d}{dt}\left(T_{hearth}\right) = A \cdot \varepsilon_1 \cdot \alpha_0 \cdot \sigma \cdot \left(T_{roof} + 273\right)^4 - A \cdot \varepsilon_0 \cdot \alpha_1 \cdot \sigma \cdot \left(T_{hearth} + 273\right)^4 - Loss_{hearth} \cdot \left(T_{hearth} - 20\right)$$

$$m_1 \cdot s_1 \cdot \frac{d}{dt}\left(T_{roof}\right) = P_{in} + A \cdot \varepsilon_0 \cdot \alpha_1 \cdot \sigma \cdot \left(T_{hearth} + 273\right)^4 - A \cdot \varepsilon_1 \cdot \alpha_0 \cdot \sigma \cdot \left(T_{roof} + 273\right)^4 - Loss_{roof} \cdot \left(T_{roof} - 20\right)$$

T_{roof} = Celcius temperature of roof

σ = Stefan-Boltzmann constant

T_{hearth} = Celcius temperature of hearth

A = Surface area of radiative exchange surfaces

α_0, ε_0 = absorptivity and emissivity of hearth

$m_0 \cdot s_0$ = mass times specific heat of hearth

α_1, ε_1 = absorptivity and emissivity of roof

$m_1 \cdot s_1$ = mass times specific heat of roof

Figure 2: Equations of the MathCAD 2000 model

4 RESULTS

Test 1 (Fig. 3) showed that with = 1, the two bodies achieve the same temperature and cool together. Test 2 (Fig. 4) showed that with $\varepsilon = 1$, super heat performance was achieved with the Aluminium preheated to 650 and the roof/elements at 820. Test 3 (Fig. 5) gave concerns with $\varepsilon < 1$, and with the Aluminium preheated to 650 and the roof/elements at 820.

Table 1: Parameters for model

Parameters for run 1	Values
Emissivity	0 to 1
Aluminium	650
Roof	820

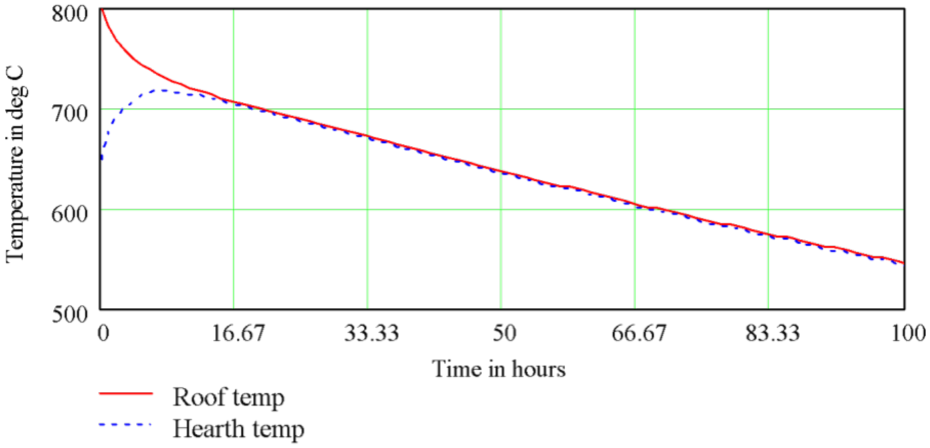

Figure 3: Power off condition

5 CONCLUSIONS

To achieve superheat performance, the roof/elements must be preheated to 820 degrees C, and the Aluminium melt must be at around 650 degrees C.

ACKNOWLEDGEMENTS

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[2] F. J. Bayley & A. B. Turner. *Heat Transfer*. Nelson (1972).
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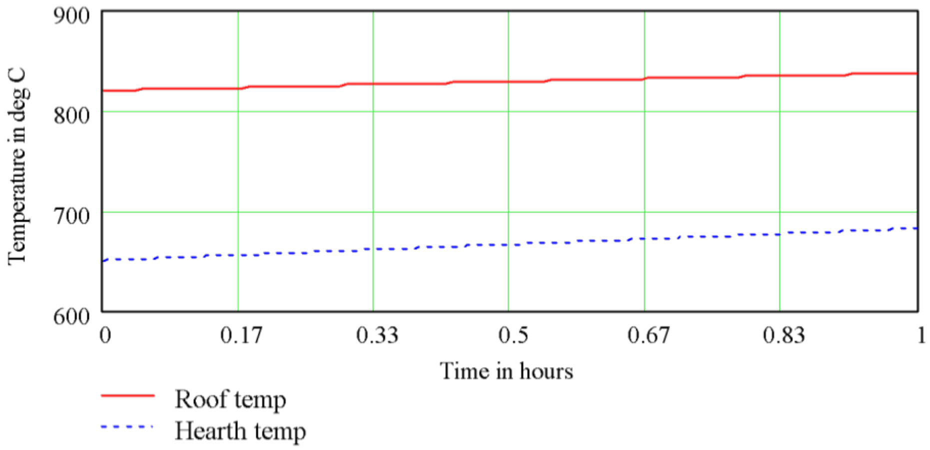

Figure 4: Predicted super heat performance with good surfaces (32 degrees C per hour)

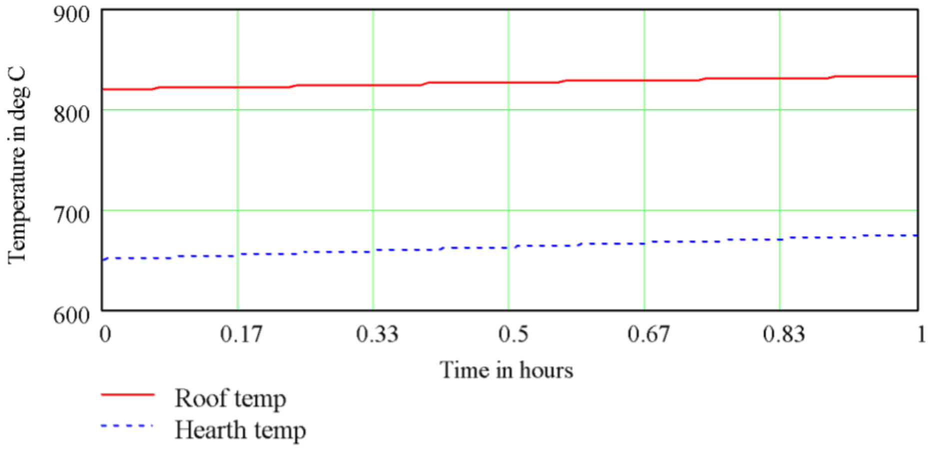

Figure 5: Predicted super heat performance with poor surfaces (24.5 degrees C per hour)