



VALIDATION OF TURBULENCE MODELS FOR FLOW AND HEAT TRANSFER OVER A BACKWARD-FACING STEP

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ABSTRACT

This paper presents comparisons of some of the most widely-used RANS turbulence models in their application to predicting mean flow and turbulent convective heat transfer in the non-equilibrium region of flow over a backward-facing step. Both OpenFOAM and STAR-CCM+ are used to test a selection of Eddy-Viscosity and Reynolds-stress transport models, including the standard and Launder-Sharma variants of the $k - \varepsilon$ model, the SST $k - \omega$ model, and the Elliptic-Blending and Launder-Reece-Rodi versions of the Reynolds stress transport model. The resulting predictions are compared with experimental data obtained in Vogel and Eaton's study of combined heat transfer and fluid dynamics downstream of a heated backwards facing step. Flow quantities such as velocity, streamwise turbulence intensity, temperature and Stanton number are compared with the experimental data obtained at a Reynolds number of 113,000. The wall-normal component of turbulent heat flux is compared to experimental data obtained at a Reynolds number of 52,000. The models tested are found to be in agreement with the experimental data in the prediction of flow reattachment, as well as flow velocity and streamwise turbulence intensity. The Launder-Sharma $k - \varepsilon$ and Elliptic Blending stress transport models are shown to predict Stanton number along the heated wall the most accurately, suggesting that low-Reynolds number models performed better at calculating wall heat transfer, than those that employ log-law-based wall functions. The merits of the addition of the Yap correction term to the Launder-Sharma model for non-equilibrium flows are also demonstrated. The Eddy-Diffusion Hypothesis is employed by each of the models tested in the calculation of turbulent heat flux. Its suitability to capturing turbulent convective heat transfer in the recirculating region is discussed.

1. INTRODUCTION

RANS modelling is the most widely used approach for the simulation of engineering turbulent flows in industry and academia due to its computational robustness. Because of their approximate nature, a diverse range of RANS approaches has been developed for the modelling of the turbulent stress tensor, which represents the effects of turbulence mixing on momentum transport. Different models involve the use of different parameters, levels of approximation and treatments of the near-wall turbulence. In turbulent heat convection, there is the additional question of how to approximate the turbulent heat flux vector, resulting in an even greater variety of modelling strategies. For relatively simple flows, use of all models results in similar and satisfactory simulations. When more complex flow phenomena are present, there are notable differences in the resulting predictions. The computation of turbulent heat convection in the presence of flow separation is a phenomenon of strong engineering importance, which still poses severe challenges to most currently used models. This study consequently investigates the predictive capabilities of RANS models in the computation of flow and heat transfer over a backward-facing step. The models tested are widely used and involve different approaches to the modelling of the turbulent stresses and the near-wall turbulence. Moreover, two different CFD codes are employed, the open source code OpenFOAM, which is gaining ground among academic researchers, and the commercial code Star-CCM+, which is widely used in industry.

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2. CASE STUDY DESCRIPTION AND SET-UP

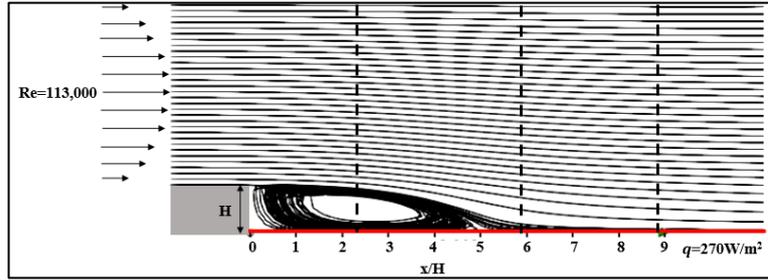


Figure 1: The computational domain illustrated by streamline data.

The chosen case study follows Vogel and Eaton's [1] experimental study of heat transfer downstream of a backward-facing step. This is a well-studied, but complex, flow regime that appears in a wide range of engineering applications. Figure 1 shows flow detachment at the step and reattachment along the downstream channel.

Flow is almost fully developed when it reaches the step, at a Reynolds number of around 110,000 for mean flow and turbulence data, and again at a Reynolds number of 52,000 for the heat flux data, based on upstream channel height. The inlet boundary condition was extracted at a point in a developing channel flow in which the boundary layer characteristics matched those from a reference point in Vogel and Eaton's [1] flow, located $4H$ upstream of the step. The upstream channel has a height of $4H$. The working fluid in this study is air, Prandtl number value of 0.71, which enters the domain at a uniform temperature before being heated by the bottom wall of the downstream channel with uniform wall heat flux thermal boundary conditions. The temperature difference across the domain is not expected to be large enough to cause expansion of the fluid, and so density is assumed constant. In this study, two different computational grids have been created. For the low-Reynolds number models, a fine grid has been created so that the first near-wall data point would lie in the highly viscous region. For the high-Reynolds number models, a coarser grid is used, so that the first near-wall grid point would lie suitably in the fully turbulent near-wall region, a condition necessary for the use of wall functions. A grid independence study has been conducted for both of the grids.

3. TURBULENCE MODELLING

The models chosen are widely available in many CFD software packages. The two model categories considered here are two-equation eddy-viscosity models (EVMs) and Reynolds stress transport models (RSMs). The EVMs employ the eddy-viscosity approximation, in which the turbulent viscosity, ν_t , is introduced to linearly relate the turbulent stress tensor to the mean strain rate tensor. Although based on a highly simplifying approximation, EVMs are often used because they are numerically robust.

In the $k - \varepsilon$ models, ν_t is determined from the turbulent kinetic energy, k and dissipation rate, ε , for which additional transport equations need to be solved. The standard $k - \varepsilon$ model [2] is a high-Reynolds-number model which employs wall functions, instead of resolving the flow across the buffer and viscous wall sub-layers. The Launder-Sharma (L-S) model [3] is a low-Reynolds number variant, meaning that all transport equations are numerically solved across the near-wall sub-layer. To improve its performance in recirculating and impinging flows, a so-called Yap correction [4] has been implemented via a source term in the dissipation equation of the Launder-Sharma $k - \varepsilon$ model.

The $k - \omega$ model [5] determines ν_t from k and specific dissipation rate, ω , which is proportional to ε/k . An advantage of $k - \omega$ is that it can be applied across the near-wall sub-layers without the need for additional terms. A model variant known as SST, 'Shear Stress Transport', $k - \omega$ [6] has been chosen for this case study. It addresses the shortcomings of the standard $k - \omega$ model when applied to freestream regions. This model is in both OpenFOAM and Star-CCM+.

Finally, we consider Reynolds stress transport models, in which the Reynolds stress tensor is computed using transport equations. These models exactly account for the advection and generation rate of the turbulent stresses. Here we test the Launder-Reece-Rodi (LRR) [7], high-Reynolds-number version, implemented in OpenFOAM, and the Elliptic Blending (EB) [8], low-Reynolds number model, as implemented in Star-CCM+.

Much like the Reynolds stress tensor, resulting from the Reynolds decomposition of the momentum equations, the turbulent heat flux vector $\overline{u_i' t'}$ appears in the Reynolds-averaged enthalpy equation and represents the transport of thermal energy by turbulent eddies. Each of the solvers in this study apply the Eddy-Diffusivity Hypothesis for the modelling of the turbulent heat fluxes, including the RSMs, which would not otherwise calculate ν_t . Based on the conduction law, the approximation assumes that turbulent heat flux in a given direction is driven only by the temperature gradient in that direction. For example, turbulent heat flux in the wall-normal, y , direction can be approximated by:

$$\overline{u_2' t'} = \frac{\nu_t}{Pr_t} \frac{\partial T}{\partial y}.$$

In reality, temperature gradients in any direction can contribute to a single component of turbulent heat flux, but for simple flows, such contributions are insignificant. The suitability of this approximation in detached and recirculating flows may be examined in the comparisons between computed turbulent heat flux and Vogel and Eaton's data.

4. RESULTS AND DISCUSSION

Table 1: Locations of flow reattachment predicted by the various OpenFOAM (O) and Star-CCM+ (S) models.

Model	Reattachment point (x/H)	%diff. compared to [1]	Reference
Experimental data	6.67	-	[1]
Standard $k - \varepsilon$ (S)	7.15	7.20	[2]
Launder-Sharma $k - \varepsilon$ (O)	6.38	4.35	[4]
SST $k - \omega$ (S)	7.37	10.49	[6]
SST $k - \omega$ (O)	7.03	5.40	[6]
Launder-Reece-Rodi RSM (O)	6.59	1.20	[7]
Elliptic Blending RSM (S)	6.48	2.85	[8]

For each of the models tested, the point at which the flow reattaches to the downstream wall is found via the wall shear-stress variation. As shown in Table 1, the RSMs predict this reattachment the most accurately, closely followed by the Launder-Sharma model with the Yap correction term.

Predictions of the velocity field show each of the models to be in good agreement with the experimental data. Figure 2 shows velocity profiles extracted at locations within the recirculation zone, just before reattachment and further downstream, where the flow recovers, as illustrated in Figure 1.

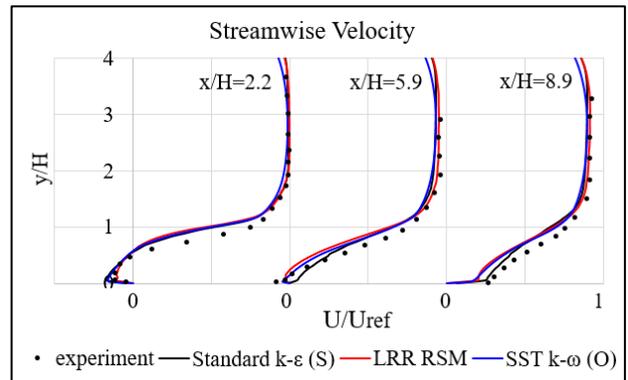


Figure 2: Velocity profiles for LRR RSM, SST $k - \omega$ and standard $k - \varepsilon$.

Figure 3 shows profiles of streamwise turbulent intensity. Both RSMs, in close agreement with each other, appear to predict turbulence intensity more accurately than the SST $k - \omega$, particularly within the recirculating region. This is generally expected for models that directly compute the Reynolds stress tensor, as opposed to employing the eddy-viscosity hypothesis. In general, the prediction of streamwise turbulence intensity is in good agreement with the experimental data.

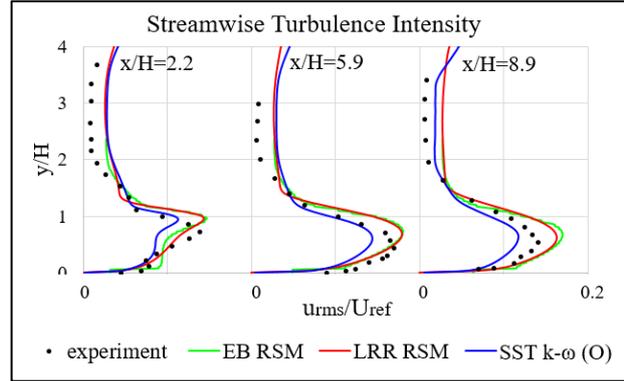


Figure 3: Streamwise turbulence intensity profiles for EB RSM, LRR RSM and SST $k - \omega$.

In terms of the mean flow and levels of turbulence, each of the models capture the behaviour of the flow with generally good accuracy. This forms a good basis to begin investigating the effect of the prediction of turbulence mixing and near-wall flow phenomena on convective heat transfer. Although there is no experimental data available for the Reynolds shear stress in Figure 4, computational data can still be used to indicate the level of turbulent mixing, and therefore the regions in which enhanced convective heat transfer can be expected.

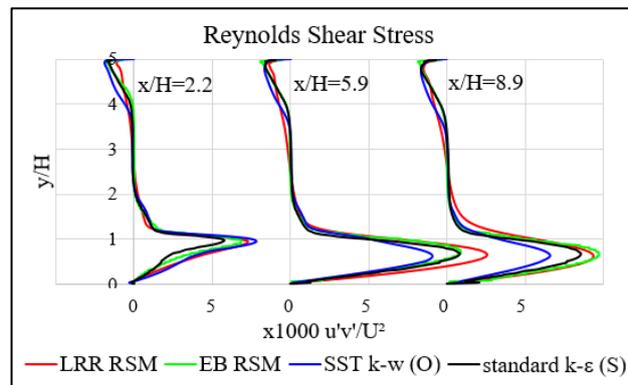


Figure 4: Reynolds shear stress distributions for LRR, EB RSM, SST $k - \omega$ and standard $k - \epsilon$.

For each profile, the Reynolds shear stress appears to peak in the region of the separated shear layer where turbulent structures in the boundary layer of the upstream channel wall are growing upon flow separation from the step. The profile at $x/H = 2.2$ indicates the separated shear layer is relatively thin and the low levels of turbulent mixing are due to the weak recirculation zone. The highest peaks can be observed around $x/H = 5.9$, which is close to the point of flow reattachment. Therefore, enhanced turbulent convective heat transfer can be expected in this region.

The above idea is supported by the data in Figure 5, in which the Stanton number also peaks around the reattachment point. The Stanton number is expected to peak in this region because the elevated levels of shear stress and turbulent kinetic energy enhance turbulent mixing and thus the transport of thermal energy away from the wall. In the recirculation region, energy transport is hindered, as the fluid heated by the wall is unable to be transported to the rest of the flow. After reattachment, the flow begins to recover, and a boundary layer redevelops to a finite thickness.

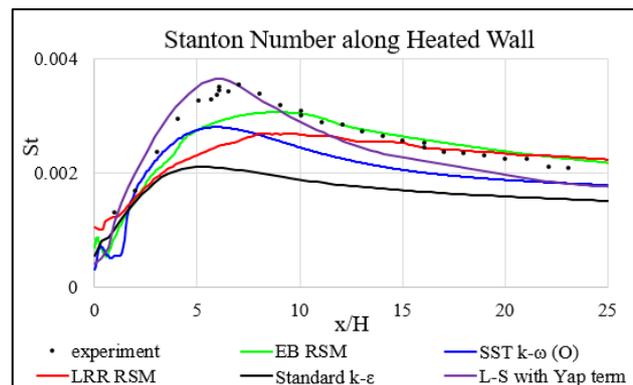


Figure 5: Stanton number distribution along the heated wall for L-S $k - \epsilon$, standard $k - \epsilon$, SST $k - \omega$ and EB RSM.

This behaviour described above is illustrated in the data in Figure 5 wherein most of the data sets show the Stanton number beginning to settle far downstream of reattachment. The Elliptic Blending RSM and the adapted Launder-Sharma model predict the Stanton number the most accurately out of all of the models tested. The data sets from the high-Re $k - \epsilon$ models lie further from the experimental data

than the other computational data sets. For the standard $k - \varepsilon$ model, it can be concluded that the use of wall functions caused this under-prediction of wall heat transfer. Near-wall quantities in the recirculating flow regions are unlikely to be well captured by log-law-based wall functions which are based on attached boundary layer theory. Although both the $k - \varepsilon$ and SST $k - \omega$ employ the effective-viscosity approximation, the Stanton number prediction of the latter is far more accurate. This highlights the predictive flaws of the log-law-based wall-functions.

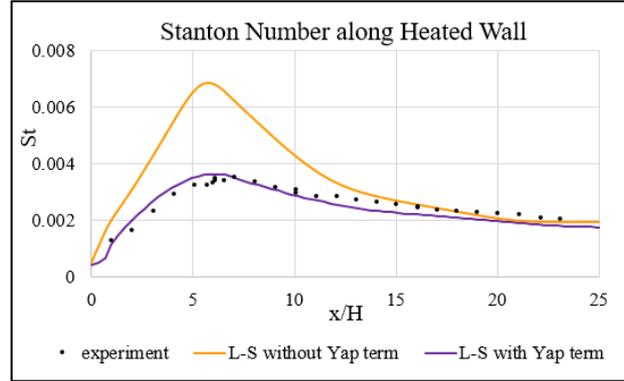


Figure 6: Stanton number comparisons for the L-S $k - \varepsilon$ model, with and without the Yap correction term.

Traditionally, turbulent length scale is over-predicted by the Launder-Sharma model in regions of non-equilibrium. Figure 6 shows the Stanton number distribution using the Launder-Sharma model readily available in OpenFOAM, and the adapted version which includes the Yap correction term. The over-prediction of turbulent length scale in the original model has presented itself in a peak Stanton number almost double that of the experimental data. In this case this implies that the over-estimation of the turbulent length scale has resulted in an under-prediction of wall temperature within the recirculation region. It is clear that the Yap correction term successfully reduces the turbulent length scale towards equilibrium values, and only in regions of non-equilibrium, as the Stanton number predictions remain in good agreement with experimental data in both the recirculating region and upon flow reattachment and recovery. The merits of the Yap correction term are therefore apparent in the prediction of near-wall flow variables in backward-facing step flows.

In Figure 7, profiles of the wall-normal turbulent heat flux, $\overline{u_2' t'}$, show relatively high levels of turbulent heat flux in the near-wall region and the separated shear layer. Both of these regions have high wall-normal velocity gradients, and therefore wall-normal temperature gradients. Each model shows this behaviour but over-predicts the magnitude of the turbulent heat flux. Each of the models tested employ the Eddy-Diffusivity Hypothesis, including the RSMs that do not compute ν_t for any reason other than to obtain α_t . This could be one of the sources of inaccuracy.

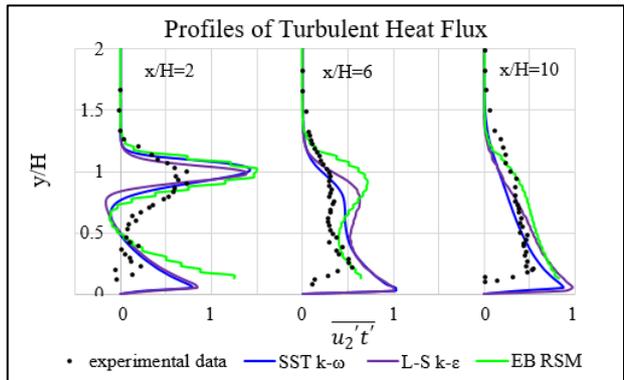


Figure 7: Wall-normal components of turbulent heat flux for SST $k - \omega$, L-S $k - \varepsilon$ and EB RSM.

Another source of uncertainty could be in the prediction of temperature gradient, particularly in the usually problematic areas of non-equilibrium, such as the separated shear layer and the recirculation zone. Future work could involve testing models that employ the Generalised Gradient Diffusion Hypothesis (GGDH), which does account for the small contributions made by temperature gradients in the other directions. However, reasonably accurate predictions of the Reynolds stress tensor components are required, so the GGDH is only appropriate for RSMs to employ in solving the energy or temperature equations.

Finally, Figure 8 shows temperature profiles inside the recirculating region, soon after reattachment, and further downstream where the flow recovers. Each model is shown to over-predict the temperature in the recirculating region, which could be a result of the Eddy-Diffusivity Hypothesis employed by each of the models. The recirculating region of flow adds a level of complexity to turbulent mixing, and this region could experience relatively higher temperature gradients in all directions of the flow, which have been considered negligible in the formulation for turbulent heat flux.

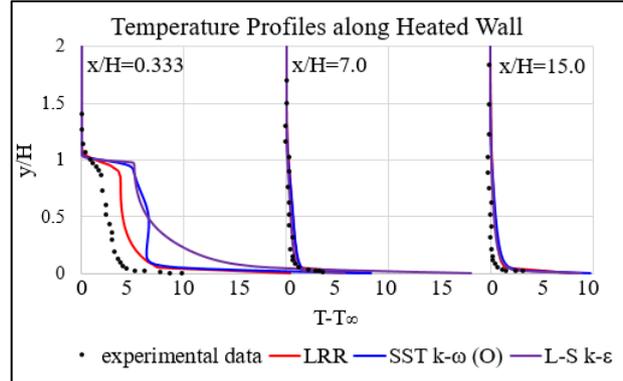


Figure 8: Temperature profiles for LRR RSM, SST $k - \omega$ and L-S $k - \epsilon$.

Additionally, the larger over-prediction of turbulent heat flux in the separated shear layer and near-wall region, seen in Figure 7, could also have caused the over-prediction of temperature in these regions, and in this case has likely affected the flow temperature between these areas.

5. CONCLUSIONS

This paper has presented some representative comparisons for some of the most widely-used turbulence models across two CFD platforms in the prediction of turbulent flow separation and reattachment, and has discussed the implications of such on the prediction of turbulent convective heat transfer. Each of the models tested has shown good agreement with experimental velocity and turbulence data. The RSMs consistently performed better in the prediction of Stanton number than the EVMs, apart from the adapted Launder-Sharma model which predicted the Stanton number the most accurately. For both EVMs and RSMs, low-Reynolds number models consistently performed better than the high-Reynolds number models where near-wall flow calculations were concerned. The implementation of the corrective Yap term in the Launder-Sharma model has been shown to dramatically improve predictions of near-wall heat transfer within the non-equilibrium region of the flow. This paper has highlighted the limitations of the Effective-Diffusivity Hypothesis in recirculating flows, demonstrating that each of the models over-predicts both temperature and turbulent heat flux in areas of higher strain. Future work on the implementation of GGDH has been suggested in order to explore the effect of accounting for all contributions of temperature gradients in the prediction of turbulent heat flux and temperature in the recirculation region of a backward-facing step flow.

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