SIMULATIONS OF NATURAL CONVECTION FOR FENE-P FLUID ON GPU CLUSTER

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Abstract. In the present study, we simulate the natural convection of FENE-P fluid within a cubic cavity by the general pressure equation on the GPU cluster. Influences of the polymer length and the Weissenberg numbers on the heat transfer are investigated, where the latter effect is observed to be significant at long polymer length. By varying the polymer length, heat transfer enhancement (HTE) and reduction (HTR) are found at moderate Ra numbers. Overall, the short polymer chain tends to cause HTE effect, and the long polymer length acts otherwise. Here, the short polymer length achieves 12% thermal enhancement over the Newtonian fluid at $\mathrm{Ra} = 10^6$.

INTRODUCTION

Adding additives into the working fluid to improve the thermal performance has been investigated over the years, and most concentrate on the Rayleigh–Bénard convection problems. However, few are studying three-dimensional natural convection with heated sidewall, an idealized geometry for many engineering applications. In the present work, the influences of the polymer length and the Weissenberg number, Wi, are considered to determine the non-Newtonian fluid's impact on the natural convection problems inside a cubic cavity at $\mathrm{Pr} = 7$ and $\mathrm{Ra} = 10^4 \sim 10^7$. Here, the velocity and pressure coupling are via the general pressure equation, and the polymer additive is modeled using the FENE-P model [1]. Simulations are conducted on the GPU cluster.

METHODOLOGY

The governing equations for simulating the thermal viscous-elastic fluid are,

$$\frac{\partial \mathrm{p}}{\partial \mathrm{t}} + \frac{1}{Ma^2}\frac{\partial u_i}{\partial x_i} = \frac{\gamma}{RePr}\frac{\partial^2 p}{\partial x_j \partial x_j} \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{b}{Re}\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{Gr}{Re^2}\cdot T + \frac{1-b}{Wi\cdot Re}\frac{\partial \tau_{ij}^p}{\partial x_j} \tag{2}$$

$$\frac{\partial T}{\partial t} + \mathrm{u_j}\frac{\partial T}{\partial x_j} = \frac{\beta}{Gr^{1/2}Pr}\frac{\partial^2 T}{\partial x_j \partial x_j} \tag{3}$$

where b is the solvent to total viscosity ratio, and τ^p is the polymer stress. The Weissenberg number (Wi) defined as $Wi = \frac{\lambda_p u_o}{L}$. Here, $\gamma = \mathrm{Pr}$ and $\mathrm{Ma} = 0.1$ are adopted in the present work, as Toutant [2] suggested.

The polymer-addition stress τ^p is obtained by solving the conformation tensor equation. This study adopts the finitely extensible non-linear elastic dumbbell model with the Peterlin's approximation (FENE-P) [3]. An elastic spring connects a pair of spherical beads polymer molecules. The polymer stress and the conformation tensor are expressed as,

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$$\tau_{ij}^{p} = \frac{\mu_p}{\lambda_p}\left[\frac{1}{1 - tr(C_{ij})/L_{max}^2} C_{ij} - \delta_{ij}\right] \quad (5)$$

$$\frac{\partial C_{ij}}{\partial t} + \frac{\partial u_k C_{ij}}{\partial x_k} = C_{ik}\frac{\partial u_j}{\partial x_k} + C_{jk}\frac{\partial u_i}{\partial x_k} - \frac{1}{Wi}\left[\frac{1}{1 - tr(C_{ij})/L_{max}^2} C_{ij} - \delta_{ij}\right] \quad (6)$$

The numerical procedure is based on the finite volume approach with a staggered grid arrangement[4], where governing equations' spatial and temporal terms are discretized using the second-order central difference scheme and the third-order TVD Runge-Kutta scheme [5]. Besides, the third-order TVD MUSCL scheme is used on convection terms of conformation tensor to improve the numerical stability. One-dimensional decomposition using GPU-Direct is adopted for the multi-GPU computation.

NUMERICAL RESULTS

This study considers the natural convection within a cubic cavity with two differentially heated opposing vertical walls (x=0 and 1). The rest of the walls are adiabatic as shown in Fig. 1. The grid density is 128^3, which symmetry non-uniform grid using hyperbolic tangent function is adopted in all directions. Fig. 2 shows the predicted temperature and velocity vector along the vertical wall bisector (y=0.5), cutting the differential heated walls for Newtonian, HTE, and HTR case at $\mathrm{Ra} = 10^6$. The addition of the polymer changes the thermal distributions and hence velocity. Compared to the Newtonian case, the contours show the enhanced and reduced vertical velocity for the HTE and HTR walls. The reduction of the HTR case's vertical velocity is due to the elevated level of the horizontal vortex, as shown in Fig. 3, which also causes the decrease of heat transfer near the wall. Finally, the influences of the Weissenberg number on the heat transfer can be seen in Fig. 4, and enhancement and reduction are observed at different polymer extension lengths.

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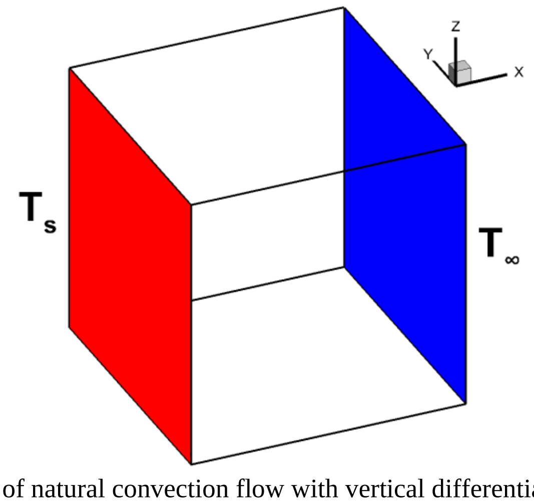
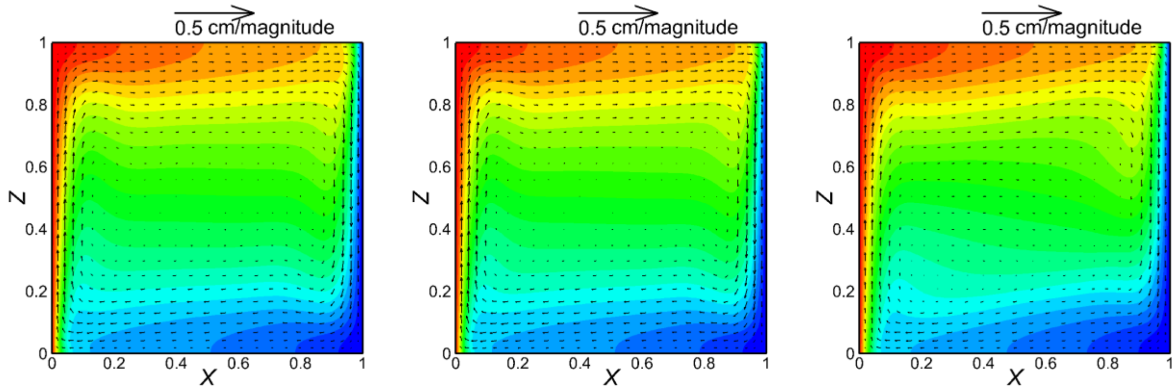
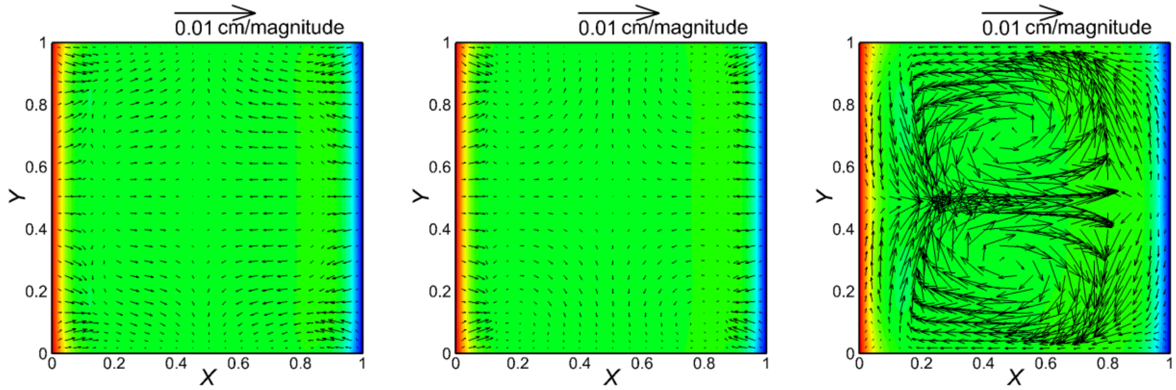


Figure 1: The geometry of natural convection flow with vertical differential heated sidewalls. The computational domain is $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$.



(a) Newtonian (b) HTE (c) HTR

Figure 2: Isotherms and velocity vector for natural convection of Newtonian fluid and non-Newtonian fluid at y=0.5 plane and $\mathrm{Ra} = 10^6$. HTE: $\mathrm{Wi} = 50$, $\mathrm{b} = 0.8$, $\mathrm{L}^2_{max} = 10$. HTR: $\mathrm{Wi} = 50$, $\mathrm{b} = 0.8$, $\mathrm{L}^2_{max} = 500$.



(d) Newtonian (e) HTE (f) HTR

Figure 3: Isotherms and velocity vector for natural convection of Newtonian fluid and non-Newtonian fluid at z=0.5 plane and $\mathrm{Ra} = 10^6$. HTE: $\mathrm{Wi} = 50$, $\mathrm{b} = 0.8$, $\mathrm{L}^2_{max} = 10$. HTR: $\mathrm{Wi} = 50$, $\mathrm{b} = 0.8$, $\mathrm{L}^2_{max} = 500$.

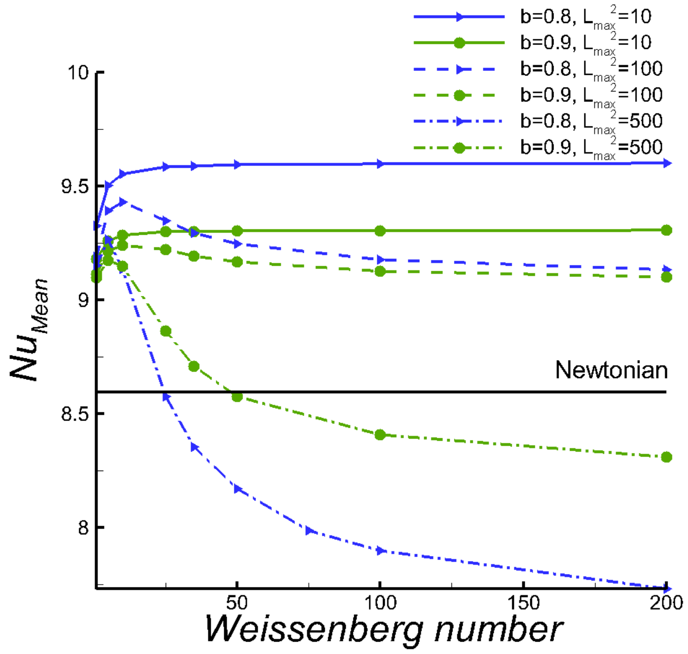


Figure 4: Mean Nusselt number distributions at different Wi numbers when $\mathrm{Ra} = 10^6$.